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Classical and Quantum Descriptions of a Nonlocal Confined Particle—Quarks

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Some aspects of classical and quantum theories of quarks, gluons, and their interaction mechanisms are considered within the framework of stochastic space-time, which is induced by a random string field. It turns out that quarks and gluons are extended objects, the propagators of which are entire functions in the momentum space. It is shown that the topological structure of space-time inside hadrons plays an essential role in quark confinement and asymptotic freedom. The strong coupling constant and confinement potential, mass, and energy of quarks depend on the Euler number, which gives rise to a unified description of the interaction picture in QCD at short and long distances. Quantization of nonlocal quark fields is achieved by using an indefinite metric, and their electromagnetic nonlocal interaction is also considered.

1. INTRODUCTION

It is well known that the majority of observable matter exists in the form of bounded states (beginning as galaxies, stars, the solar system, atoms, and finishing as hadrons).

Forces responsible for holding these forms of matter, except hadrons, are of gravitational and electromagnetic nature, which decrease as $1/r^2$, where r is the distance between the constituent parts of the bounded states. The nature and type of the force between the quarks from which the hadron structure is formed are not well known, even in the framework of quantum chromodynamics (QCD). One reason is that QCD does not work at large distances. The confinement properties of quarks and gluons give rise to

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the proposition that the force between them increases as r^n, n being some phenomenological constant that should be defined from experimental data.

QCD is the modern theory of strong interactions, where quarks and gluons carrying the "color" quantum number play the essential role in the understanding of the hadron structure, at least in the short-distance regime (asymptotic freedom).

However, we still lack a good understanding of longer distance hadron physics, in particular when quarks and gluons are "confined" inside hadrons. Many attempts (Efimov and Ivanov, 1993) have been made in this direction, giving some hope of success.

A distance-dependent regime of QCD makes sense in that not only does the strong interaction picture depend on the energy scale (distance) (for example, the strong coupling "constant" decreases with increasing momentum), but also constituent quarks and gluons carrying this interaction may possess some unusual inner properties that depend on this scale. In this paper, we will try to understand quarks and gluons as composite objects connected with the space-time structure in their neighborhood (or inside hadrons). We assume that space-time, due to the presence of quarks, begins to fluctuate inside the hadrons and its topological structure gives rise to changes in the physical quantities of quarks.

This means that the structure of space-time is distorted in the immediate neighborhood of a particle (quark), which leads to the concept of confinement and to the problem of reformulating the dynamics of the particle. Further, we propose that random stringlike objects carry a fluctuational property of space-time and that these stochastic strings alter the geometry of space-time (at least inside hadrons); this alteration, in turn, affects the behavior of quarks. In Namsrai (1993) we showed that this self-referential nonlinear property of gravity is responsible for the appearance of the nonlocal interaction of quarks and the confinement force between them. It turns out that quarks and the force-transmitting quanta—gluons—become extended objects, the propagators of which possess unusual properties. Moreover, the structural aspect of this nonlocality depends on the energy (distance) of the quarks and gluons. Roughly speaking, this distance-dependent property may be understood as follows. When quarks and gluons travel faster, they look like pointlike objects (at small distances) and interaction between them takes place at a point, i.e., the local-interaction picture is valid. When they move slowly, they become nonlocal or spread out over space-time, and their interaction turns out to be nonlocal. In our approach, the topological structure of space-time inside hadrons plays an essential role. Roughly speaking, if we assume that hadrons are spread out in a domain characterized by the parameter $\sqrt{\alpha'_0} \sim m_p^{-1}$ and are stringlike carriers of these structural properties with respect to space-time, then all physical quantities of their constituent parts—quarks—depend on a topological invariant known as the Euler characteristic of a two-dimensional surface M by which hadrons are covered.

For example, the mass of a quark depends not only on its velocity as $m = m_0(1 - v^2/c^2)^{-1/2}$, in accordance with the special theory of relativity, but it also depends on the Euler number

$$m = \frac{m_0}{(1 - v^2/c^2)^{1/2}} \exp\left\{\frac{\lambda^2}{2\alpha'} \operatorname{Euler}(M)\right\}$$

Here λ is some constant with the dimension of length (see below). If M has genus N (i.e., M is homeomorphic to a sphere with N handles, or to a connected sum of N tori), then $\operatorname{Euler}(M) = 2 - 2N$. Moreover, as shown below (Section 2), the potential force between quarks and the strong coupling constants also depend on the topological structure of space-time inside hadrons:

$$g = g_0 \exp\left\{\frac{\lambda^2}{2\alpha'} \operatorname{Euler}(M)\right\}$$

We observe an interesting fact if we take into account the Lorentz factor in the definition of the parameter $\alpha' = \alpha_0'(1 - Q^2/c^2)$, then, due to the presence of the topological multiplier (with $N \geq 2$), the strong coupling "constant" decreases with increasing momentum Q of hadrons ("runs"), precisely as predicted in 1973 with the discovery of asymptotic freedom. Thus asymptotic freedom, one of the pillars of QCD, is caused by the topology of space-time at short distances.

This fact explains why, with respect to topology, hadrons may be considered as a connected sum of $N \geq 2$ tori; the mass of the quark and its coupling constant at low energies go to zero when $N \to \infty$. The latter case gives some hope of a unified description of the short- and longer distance effects by means of the topological structure of hadrons.

The purpose of this paper is to study some aspects of classical and quantum theories of nonlocal "confined" particles, to carry out the quantization procedure by means of indefinite metrics, and to show the finiteness of the theory. Section 2 is devoted to formulating the problem within the framework of the induced gravitation due to stochastic string fields. In Section 3 we consider the classical motion of the particle that is "confined" in some domain characterized by the parameter α_0', where $\alpha_0' \sim m_p^{-2}$ is the inverse string-nucleon tension (or its size).

In Section 4 we briefly discuss the quantum mechanical consideration of "confined" particles and obtain their Schrödinger equation. Section 5 deals with the Lagrangian formulation of extended quarks and its regularization and quantization procedures, in accordance with the Efimov (1977) nonlocal theory. The construction of the finite gauge-invariant S-matrix and the calculation of some of the primitive Feynman-type diagrams of the nonlocal electromagnetic interaction of quarks are carried out in Section 6. Within the framework of our scheme, the nonlocal interaction Lagrangian of QCD, concrete calculation of the S-matrix elements, and proof of finiteness and gauge invariance of the theory for such a Lagrangian will be studied elsewhere.

2. STOCHASTIC STRINGS AND TOPOLOGICAL STRUCTURE OF SPACE-TIME INSIDE HADRONS

Our main assumption is that hadrons are spread out in some domain, in the immediate neighborhood of which the space-time structure is distorted and fluctuating. This structure is described by a random string field having the appearance of a surface effect with respect to the empty space-time outside the hadrons. In other words, stochastic strings alter the geometry of space-time, and that alteration in turn affects the behavior of particles—hadrons. It turns out that the self-referential nonlinear property of gravity is responsible for the appearance of the nonlocal interaction and the confinement force between quarks, and their coupling constant depends on the hadron momentum and the Euler number of the "hadron" surface, which may be chosen arbitrarily small in both the short- and long-distance regimes of QCD.

Let us consider the stochastic space-time induced by random strings, the behavior of which is described by the probability distribution

$$P[Y] = \frac{1}{N}\exp\left\{-\frac{1}{2}\int_{M_1}\int_{M_2} d^2\sigma_1\, d^2\sigma_2\, \sqrt{g_1}\sqrt{g_2}Y^\mu(\sigma_1)D^{-1}_{\mu\nu}(\sigma_1-\sigma_2)Y^\nu(\sigma_2)\right\} \tag{1}$$

where M is a two-dimensional surface known as the string world-sheet ($\sigma = \sigma^a = \{\sigma^1 = \sigma, \sigma^2 = \tau\}$), which is equipped with a metric tensor g_{ab}, while space-time has coordinates $x^\mu(\tau)$ and metric $G_{\mu\nu}$; $Y^\mu(\sigma)$ are the coordinates of the strings, and N is a normalization constant.

$D^{-1}_{\mu\nu}$ is the inverse of the two-point correlation

$$\langle Y^\mu(\sigma_1)Y^\nu(\sigma_2)\rangle_Y = D^{\mu\nu}(\sigma_1-\sigma_2) \tag{2}$$

In this paper we use the white noise covariance

$$D^{\mu\nu}(\sigma_1-\sigma_2) = -\eta^{\mu\nu}\frac{\lambda^2}{\sqrt{g}R}\delta^2(\sigma_1-\sigma_2) \tag{3}$$

Here R is the Ricci curvature scalar of the manifold M, and λ is some constant with the dimension of length. We distinguish two possibilities: (a) $\lambda^2 \sim G$, where G is the Newtonian constant; (b) $\lambda^2 \sim \alpha_0'$, where α_0' is the inverse string tension (the size of the string).

The former means that fluctuations of the string coordinate takes place at the Planck scale, while the second case means that coordinates $Y^\mu(\sigma)$ obey random properties in a domain characterized by the size of the hadron. Further, we propose that coordinates $Y^\mu(\sigma)$ are made to fluctuate around the usual space-time coordinate $x^\mu(\tau)$:

$$Y^\mu(\sigma) = \frac{1}{\sqrt{g}R}\, \delta(\sigma^1)x^\mu(\sigma^2 = \tau) + \eta^\mu(\sigma^1, \sigma^2) \tag{4}$$

where the functions $\eta^\mu(\sigma)$ are random variables of type of $Y^\mu(\sigma)$.

As shown in Namsrai (1993), the formal transformation

$$\xi^\mu \Rightarrow x^\mu \exp\left\{\frac{1}{2(\pi\alpha')^{1/2}} \int_M d^2\sigma\ \sqrt{g}Rn_\mu(\sigma)Y^\mu(\sigma)\right\} = x^\mu\Lambda$$

[$n_\mu(\sigma)$ is a unit vector, $-\eta^{\mu\nu}n_\mu(\tau)n_\nu(\tau) = 1$, depending on the timelike variable $\sigma^2 = \tau$ only] leads to the metric tensor

$$G_{\mu\nu}(x, Y) = \Lambda^2[\eta_{\mu\nu} + \epsilon_{\mu\nu}(x) + \tfrac{1}{4}\epsilon^\rho_\mu(x)\epsilon_{\nu\rho}(x)] \tag{5}$$

and

$$G^{\nu\sigma}(x, Y) = \Lambda^{-2}[\eta^{\nu\sigma} - \epsilon^{\nu\sigma}(x) + \tfrac{3}{4}\epsilon^{\nu\rho}(x)\epsilon^\sigma_\rho(x) - \cdots]$$

where

$$\epsilon^\alpha_\mu(x) = \frac{1}{\sqrt{\pi\alpha'}}\, x^\alpha(\tau)n_\mu(\tau)$$

It is easily verified that

$$G^{\nu\sigma}(x, Y)G_{\mu\nu}(x, Y) = \delta^\sigma_\mu$$

Now let us carry out an averaging procedure in (5) over the random variable $Y^\mu(\sigma)$ with the probability distribution (1). Taking into account formulas (3) and (5) and using the Feynman rules (Feynman and Hibbs, 1965), we have

$$\begin{aligned} G_{\mu\nu}(x) &= \langle G_{\mu\nu}(x, Y)\rangle_Y \\ &= \left[\eta_{\mu\nu} + \epsilon_{\mu\nu}(x) + \frac{1}{4}\, \epsilon^\rho_\mu(x)\epsilon_{\nu\rho}(x)\right] \\ &\quad \times \exp\left\{\frac{\lambda^2}{2\pi\alpha'} \int_M d^2\sigma\ \sqrt{g}R\right\} \end{aligned} \tag{6}$$

Here we have used the differential of the string coordinates

$$\frac{\partial Y^{\mu}(\sigma)}{\partial x^{\nu}(\tau)} = \delta^{\mu}_{\nu} \frac{1}{\sqrt{g}R} \delta(\sigma^1)\delta(\sigma^2 - \tau)$$

in accordance with formula (4) and the fact that $n_{\mu}(\tau)$ is a unit vector. By definition,

$$\text{Euler}(M) = \frac{1}{4\pi} \int_M d^2\sigma \sqrt{g}R = 2 - 2N$$

where it is assumed that M has genus N (i.e., if M is homeomorphic to a sphere with N handles, or to a connected sum of N tori); one gets

$$G_{\mu\nu}(x) = \exp\left\{\frac{4\lambda^2}{\alpha'}(1 - N)\right\}\left[\eta_{\mu\nu} + \epsilon_{\mu\nu}(x) + \frac{1}{4}\epsilon^{\rho}_{\mu}(x)\epsilon_{\nu\rho}(x)\right] \tag{7}$$

This is a main formula in our scheme. Formula (7) means that the physical space-time metric of the hadron matter is slightly changed with respect to the Minkowski one due to topological properties of space-time at short distances (or inside hadrons).

Let us study the constituent parts—quarks—in hadrons by means of the space-time metric (7). First of all, it should be noted that when a hadron travels with velocity $\mathbf{Q}$, then its proper "size" α' undergoes the Lorentz space contraction

$$\alpha' = \alpha'_0(1 - Q^2/c^2) \tag{8}$$

At the same time, the constituent quark carries some fraction of the total proton momentum $\chi v^2 = Q^2$; therefore the topological factor in (7) takes the form

$$\exp\left\{\frac{4\lambda^2}{\alpha'}(1 - N)\right\} = \exp\left\{\frac{4\lambda^2}{\alpha'_0[1 - (\chi v^2/c^2)]}(1 - N)\right\} \tag{9}$$

where $\mathbf{v}$ is the velocity of the quark. Thus, we are now ready to construct the theory of induced "gravity" with the metric tensor (5) or (7), by using the general covariant method (Weinberg, 1972) with respect to the system of reference x^{μ}. For example, in the limiting case when the velocity of the quark is small, an additional nonrelativistic "potential" φ appears (Landau and Lifschitz, 1971; Namsrai, 1991):

$$\varphi = \tfrac{1}{2}c^2(-1 - G_{\infty}) \tag{10}$$

Omitting the unessential constant and choosing as unit vector $n_{\mu}(\tau)$ in (5) the four-velocity ($Q_{\mu} = \sqrt{\chi}v_{\mu}$) of the hadron, we have

$$U = m\varphi = -\frac{m}{2}c^2G_\infty$$
$$= \frac{mc^2}{2}\exp\left\{\frac{4\lambda^2}{\alpha_0'}\left(1 + \frac{\chi \mathbf{v}^2}{c^2}\right)(1 - N)\right\}\left[1 - \frac{\mathbf{x}^2}{x_0^2(1 - \chi \mathbf{v}^2/c^2)}\right] \quad (11)$$

where $x_0^2 = 4\pi\alpha_0'$. Further, expanding this expression into power series of $\chi v^2/c^2$ and keeping the main terms, one gets

$$U = \frac{mc^2}{2}\left\{\rho\left(1 - \frac{x^2}{x_0^2}\right) + \frac{v^2}{c^2}\left[-2\chi\rho + L\left(1 - \frac{x^2}{x_0^2}\right)\right]\right\} \quad (12)$$

where

$$\rho = \exp\left\{\frac{4\lambda^2}{\alpha_0'}(1 - N)\right\}, \qquad L = \chi\rho\left\{2 + \frac{4\lambda^2}{\alpha_0'}(1 - N)\right\}$$

To illustrate the explicit dependence of this potential on the topological invariant N, we choose $v^2/c^2 = 1/9$, $\chi = 3$, and $\lambda^2 = \alpha_0'$.

For this particular case, the form of the quark potential is sketched in Fig. 1 for different values of N.

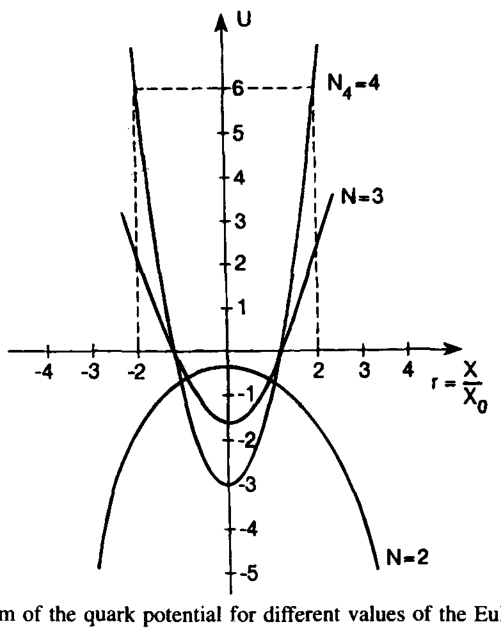

Fig. 1. Form of the quark potential for different values of the Euler number.

The second important characteristic of the quark is its energy. When the quark moves inside the hadron, its averaged energy is defined as (Landau and Lifschitz, 1971; Namsrai, 1991)

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \sqrt{-G_\infty}$$

$$= \frac{mc^2}{\sqrt{1 - v^2/c^2}} \exp\left\{\frac{\lambda^2}{\alpha_0(1 - \chi v^2/c^2)}(1 - N)\right\}$$

$$\times \left[1 - \frac{\mathbf{x}^2}{x_0^2[1 - (\chi v^2/c^2)]}\right]^{1/2} \tag{13}$$

From this we immediately conclude that the quark undergoes a finite motion, the phase diagram (Fig. 2) of which takes the form

$$\frac{\mathbf{p}^2}{p_{\max}^2} + \frac{\mathbf{x}^2}{x_{\max}^2} \leq 1 \tag{14}$$

where $p_{\max} = mc$, $\mathbf{p} = \sqrt{\chi} m\mathbf{v}$, and $x_{\max} = x_0 = 2\sqrt{\pi\alpha_0'}$. Assuming $\alpha_0' = m_p^{-2}$, we get $x_{\max} = 10^{-13}$ cm.

From expressions (11) and (14) one can conclude that the quark mass is changed according to the formula

$$m \to m_q = m \exp\left\{\frac{\lambda^2}{\alpha_0[1 - (\chi v^2/c^2)]}(1 - N)\right\} \tag{15}$$

It is easy to see that the rest mass of the quark becomes zero at sufficiently large values of N. In the nonrelativistic limit, expression (13) takes the form

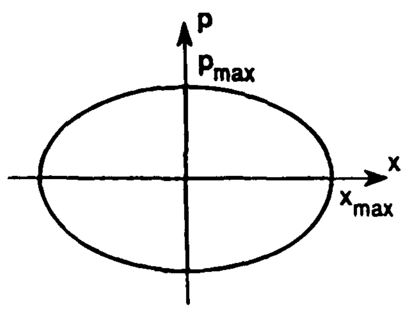

Fig. 2. The phase diagram of the quark inside a hadron.

$$E_{\text{nonrel}} = \rho'\left\{mc^2\sqrt{1 - \frac{x^2}{x_0^2}} + \frac{mv^2}{2}\left[\left(1 + 2\chi + \frac{2\chi\lambda^2}{\alpha_0}(1 - N)\right)\right.\right.$$
$$\left.\left.\times\sqrt{1 - \frac{x^2}{x_0^2}} - \frac{2\chi}{\sqrt{1 - (x^2/x_0^2)}}\right]\right\} \tag{16}$$

$\rho' = \exp\{(\lambda^2/\alpha_0')(1 - N)\}$. Here we have assumed that $x^2/x_0^2 < 1$.

Quantities (12) and (16) are quite sufficient to consider the classical motion of the quark inside the hadron by means of the topological invariant. We now go on to this problem.

3. CLASSICAL MOTION OF THE QUARK INSIDE THE HADRON

In order to illustrate the quark motion from the point of view of the classical theory we assume that a quarklike particle undergoes a finite motion in some domain, and that its potential and energy are given by formulas (12) and (16). Then, as usual, a classical description of the quark motion is carried out with the aid of the correspondence principle. The Lagrangian function of the quark has the form

$$\mathcal{L} = E_{\text{nonrel}} - U = D(x^2) + \frac{mv^2}{2}Q(x^2) \tag{17}$$

where

$$D(x^2) = mc^2\left[\rho'\left(1 - \frac{x^2}{x_0^2}\right)^{1/2} + \frac{1}{2}\rho\left(1 - \frac{x^2}{x_0^2}\right)\right]$$
$$Q(x^2) = \left(1 - \frac{x^2}{x_0^2}\right)^{-1/2}\left[(\rho' + L')\left(1 - \frac{x^2}{x_0^2}\right) - 2\chi\rho'\right] + 2\chi\rho - L\left(1 - \frac{x^2}{x_0^2}\right) \tag{18}$$

with

$$\rho = \exp\left\{\frac{4\lambda^2}{\alpha_0'}(1 - N)\right\}, \qquad L' = 2\chi\rho'\left[1 + \frac{\lambda}{\alpha_0'}(1 - N)\right],$$
$$L = \chi\rho\left[2 + \frac{4\lambda^2}{\alpha_0'}(1 - N\right]$$

The Euler–Lagrange equation

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\mathbf{v}} - \frac{\partial\mathcal{L}}{\partial\mathbf{x}} = 0$$

leads to the following equation of motion of the quark:

$$m\dot{\mathbf{v}} = Q^{-1}(x^2)\left\{\frac{\mathbf{x}}{x_0^2}\left[mc^2\left(\rho - \rho'\left(1 - \frac{x^2}{x_0^2}\right)^{1/2}\right) - \frac{mv^2}{2}H(x^2)\right]\right. \tag{19}$$

$$\left. + x_0^{-2}\mathbf{v} \times \mathbf{M}H(x^2)\right\}$$

where $M = \mathbf{r} \times \mathbf{p}$ is the angular momentum, and

$$H(x^2) = (1 - x^2/x_0^2)^{-3/2}\left[-(\rho' + L')(1 - x^2/x_0^2) - 2\chi\rho'\right] + 2L \tag{20}$$

We see that equation (19) is nonlinear and quite complicated. Such a composite form of the equation of the quark motion is caused by a self-referential effect depending on the topological structure of space-time inside the hadron. Terms in the right-hand side of (19) are responsible for a spiral-like finite motion of the quark in the domain characterized by the parameter α_0'. We now use another method for the integration problem (Landau and Lifschitz, 1965).

The point is that our problem is reduced to that of the quark motion in the external central field (12), in which its potential energy depends only on the distance r from a definite fixed point (hadron "center"). In this case, there is no need to write equation (19) of the quark motion and, by using the conservation of its energy and momentum, one can obtain a full solution of the given problem. Thus, introducing the polar coordinates (φ, r) in the Lagrangian function (17), we see that the coordinate φ is absent from it, and therefore the corresponding generalized momentum

$$P_\varphi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = mr^2 Q(r^2)\dot{\varphi}$$

is the motion integral, i.e.,

$$mr^2\dot{\varphi}Q(r^2) = M = \text{const} \tag{21}$$

From this we define $\dot{\varphi}$ through M and substitute it into the expression for the energy, obtaining

$$E_{\text{tot}} = \frac{m}{2}(\dot{r}^2 + r^2\dot{\varphi}^2)N(r^2) + D(r^2) = \left[\frac{m\dot{r}^2}{2} + \frac{M^2}{2mr^2Q^2(r^2)}\right]N(r^2) + D(r^2) \tag{22a}$$

where

$$N(r^2) = \left(1 - \frac{r^2}{r_0^2}\right)^{-1/2}\left[(\rho' + L')\left(1 - \frac{r^2}{r_0^2}\right) - 2\chi\rho'\right] - 2\chi\rho + L\left(1 - \frac{r^2}{r_0^2}\right) \tag{22b}$$

and the functions $D(r^2)$ and $Q(r^2)$ are given by formula (18). From (22) one gets

$$\dot{r} = \frac{dr}{dt} = \sqrt{\frac{2}{m}}\left[N^{-1}(r^2)(E - D(r^2)) - \frac{M^2}{2mr^2Q^2(r^2)}\right]^{1/2} \tag{23}$$

or, separating variables and integrating, we obtain

$$t = \sqrt{\frac{m}{2}}\int\left[N^{-1}(r^2)(E - D(r^2)) - \frac{M^2}{2mr^2Q^2(r^2)}\right]^{-1/2} dr + c \tag{24}$$

Further, writing (21) in the form

$$d\varphi = \frac{M}{mr^2Q(r^2)}\, dt$$

and substituting dt from (23) and integrating, we find

$$\varphi = \frac{1}{\sqrt{2m}}\int \frac{(M/r^2)N^{1/2}(r^2)}{Q(r^2)[E - D(r^2) - N(r^2)M^2/2mr^2Q^2(r)]^{1/2}} + c \tag{25}$$

Formulas (24) and (25) solve the given problem in a general form. The latter determines the connection between r and φ, i.e., the equation of the trajectory. Formula (24) defines, in a nonexplicit form, the distance r of the moving quark's point from the center as a function of time. Notice that the angle φ always changes monotonically with time since from

$$M = mr^2\dot{\varphi}Q(r^2) = \text{const}$$

it follows that $\dot{\varphi}$ never changes its sign for appropriate choices of the function $Q(r^2)$ in (18).

The radial part of the motion may be considered as one-dimensional motion in the field with the "effective" potential energy

$$U_{\text{eff}} = D(r^2) + \frac{M^2N(r^2)}{2mr^2Q^2(r^2)}$$

Values of r at which the equality

$$E = D(r^2) + \frac{M^2N(r^2)}{2mr^2Q^2(r^2)}$$

holds define a boundary of the scattering domain of the motion from center. From the physical point of view, in the general case, the variation of r has two boundaries, $r_{\text{max}} = r_0$ and $r_{\text{min}} = 0$, and therefore the motion of the quark is finite and its trajectory is wholly situated inside the circle of radius $x_0 =$

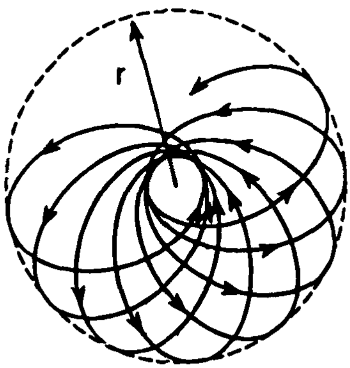

Fig. 3. Possible form of the trajectory of the quark motion inside a hadron.

$r_0 = 2\sqrt{\pi\alpha_0'}$. However, this does not mean that the trajectory is certainly a closed curve.

When r changes from $r_{max} = r_0$ to $r_{min} = 0$ and then to r_{max} again, the radius vector turns through the angle $\Delta\varphi$

$$\Delta\varphi = 2\int_{r_{min}}^{r_{max}} \frac{(M/r^2)N^{1/2}(r^2)}{Q(r^2)[2m(E - D(r^2)) - M^2N(r^2)/r^2Q^2(r)]^{1/2}} \tag{26}$$

in accordance with (25).

The condition of the closedness of the trajectory consists in that this angle be equal to the rational part of 2π, i.e., it have the form $\Delta\varphi = 2\pi m/n$, where n, m are integers. Thus, after n repetitions, the period of the radius vector of the quark point has accomplished m full rotations and coincides with its initial value, i.e., the trajectory is closed. However, such cases are rare and for suitable choices of the form of the functions $D(r^2)$, $N(r^2)$, and $Q(r^2)$ (i.e., the Euler number should be chosen arbitrarily), the angle $\Delta\varphi$ is not a rational part of 2π. Therefore in the general case the trajectory of the finite motion is not closed. It goes through its minimum and maximum distances innumerable times (as shown in Fig. 3) and fills, during an infinite time, the whole domain (i.e., inside the hadron) determined by the parameter α_0'.

4. QUANTUM MECHANICAL CONSIDERATION OF THE QUARK MOTION INSIDE THE HADRON

A quantum mechanical description of the quark motion inside the hadron is carried out in the standard way. In accordance with formula (13), the

Hamiltonian function of the nonrelativistic motion of the "free" quark takes the form

$$H = \left(m_q c^2 + \frac{m_q v^2}{2}\right)\left[1 - \frac{\mathbf{x}^2}{X_0^2}\right]^{1/2}$$

or

$$\frac{H}{(1 - x^2/X_0^2)^{1/2}} = m_q c^2 + \frac{m_q \mathbf{v}^2}{2} \tag{27}$$

where the quark mass m_q is given by expression (15) and $X_0^2 = x_0^2(1 - Q^2/c^2)$, $x_0^2 = 4\pi\alpha_0'$, and $\mathbf{Q}$ is the hadron momentum (velocity).

If the hadron is at rest, then $\mathbf{Q} = 0$. Now omitting the inessential constant $m_q c^2$ and passing to the quantum mechanical quantities, one gets

$$\frac{\hat{H}}{(1 - x^2/X_0^2)^{1/2}} = \frac{\hat{p}^2}{2m} \tag{28}$$

The usual substitutions $\hat{H} \Rightarrow i\hbar\, \partial/\partial t$ and $\hat{p} = -i\hbar\, \partial/\partial \mathbf{x}$ give

$$\left(1 - \frac{\mathbf{x}^2}{X_0^2}\right)^{-1/2} i\hbar \frac{d\Psi}{dt} = -\frac{\hbar^2}{2m} \nabla^2 \Psi \tag{29}$$

In the stationary case

$$\Psi = \text{const} \cdot e^{(-i/\hbar)Et} \Psi(\mathbf{x})$$

we have

$$\frac{\hbar^2}{2m} \nabla^2 \Psi + \left(1 - \frac{\mathbf{x}^2}{X_0^2}\right)^{-1/2} E\Psi = 0 \tag{30}$$

It is easily verified that the function

$$\Psi(x) = \text{const} \cdot \sqrt{1 - \frac{\mathbf{x}^2}{X_0^2}} \exp\left\{\frac{i}{\hbar} \mathbf{px}\left(1 - \frac{\mathbf{x}^2}{X_0^2}\right)^{-1/4}\right\} \tag{31}$$

satisfies equation (30) up to $O(1/X_0^2)$.

Thus, the full wave function of the stationary states of the quark has the form

$$\Psi = \text{const} \cdot \sqrt{1 - \frac{\mathbf{x}^2}{X_0^2}} \exp\left\{-\frac{i}{\hbar} Et + \frac{i}{\hbar} \mathbf{px}\left(1 - \frac{\mathbf{x}^2}{X_0^2}\right)^{-1/4}\right\} \tag{32}$$

$(E = p^2/2m)$.

We see that this solution correctly represents the general physical picture of the given motion of the quark. Indeed, the wave function (32) decreases and goes to zero at the "surface" of the hadron $\mathbf{x}^2 = X_0^2$ and rapidly oscillates. It has the form shown in Fig. 4.

Now we study the general situation when the constituent quark carries some fraction of the total momentum $\chi\mathbf{v}^2 = \mathbf{Q}^2$.

In this case, the total Hamiltonian function has the form (22a),

$$H = D(x^2) + \frac{mv^2}{2} N(x^2)$$

or, in quantum mechanical operator language,

$$\frac{i\hbar(d\Psi/dt) - D(x^2)\Psi}{N(x^2)} = -\frac{\hbar^2}{2m}\Delta\Psi$$

where the functions $D(x^2)$ and $N(x^2)$ are defined by formulas (18) and (22b), respectively. Since the functions $D(x^2)$ and $N(x^2)$ depend on the radius vector, $\mathbf{x}^2 = \mathbf{r}^2$, and therefore the problem is reduced to the Schrödinger equation for the quark motion in the central-symmetric potential (Landau and Lifschitz, 1963). In the stationary case, it takes the form

$$\Delta\Psi + \frac{2m}{\hbar^2}[E' - U'(r)]\Psi = 0 \tag{33}$$

where $E' = EN^{-1}(r^2)$, $U'(r) = D(r^2)/N(r^2)$.

Making use of the well-known expression for the Laplacian operator in spherical coordinates, we get

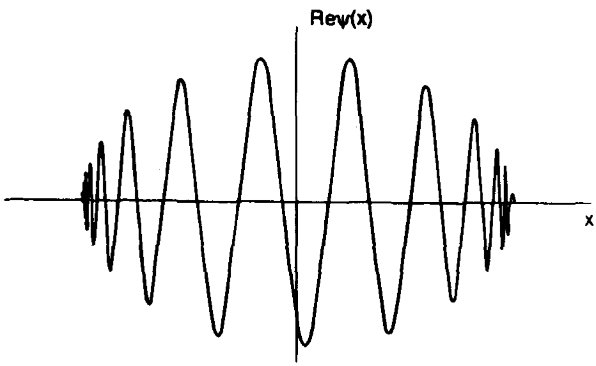

Fig. 4. General picture of the wave function of the quark inside a hadron.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\Psi}{\partial\varphi^2}\right]$$
$$+ \frac{2m}{\hbar^2}[E' - U'(r)]\Psi = 0 \tag{34}$$

If we introduce the operator

$$\hat{l}^2 = -\left[\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\right]$$

of the angular momentum, then we obtain

$$\frac{\hbar^2}{2m}\left[-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right) + \frac{\hat{l}^2}{r^2}\Psi\right] + U'(r)\Psi = E'\Psi \tag{35}$$

For the motion in the central-symmetric field, the angular momentum is conserved. We will consider stationary states in which $\hat{l}^2$ and l_z have definite values. In other words, we find general eigenfunctions of the operators $\hat{H}$, $\mathbf{l}^2$, and $\hat{l}_z$.

The condition that Ψ is an eigenfunction of the operators $\hat{\mathbf{l}}^2$ and $\hat{l}_z$ is that it defines its angular dependence. Therefore we seek a solution of equation (35) of the form

$$\Psi = R(r)Y_{lm}(\theta, \varphi) \tag{36}$$

where $Y_{lm}(\theta, \varphi)$ are given by the standard expression

$$Y_{lm}(\theta, \varphi) = \frac{1}{\sqrt{2\pi}} e^{im\varphi}\theta_{lm}(\theta, \varphi)$$
$$= \frac{1}{\sqrt{2\pi}} e^{im\varphi}(-1)^m i^l \sqrt{\frac{(2l+1)}{2}\frac{(l-m)!}{(l+m)!}} P_l^m(\cos\theta) \tag{37}$$

Further, taking into account the identity

$$\hat{l}^2 Y_{lm} = l(l+1)Y_{lm}$$

we have the following equation for the "radial functions" $R(r)$:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - \frac{l(l+1)}{r^2}R + \frac{2m}{\hbar^2}[E' - U'(r)]R = 0. \tag{38}$$

The standard substitution $R(r) = \chi(r)/r$ reads

$$\frac{d^2\chi}{dr^2} + \left[\frac{2m}{\hbar^2}(E' - U') - \frac{l(l+1)}{r^2}\right]\chi = 0 \tag{39}$$

The physical requirement of quark confinement gives

$$\chi(r = r_0) = 0$$

and

$$R \Rightarrow R_{kl} = (-1)^l \sqrt{\frac{2}{\pi}} \frac{r^l}{k^l} \left(\frac{d}{r\,dr}\right)^l \frac{\sin kr}{r}, \qquad k = \frac{p}{\hbar} = \frac{\sqrt{2mE}}{\hbar}$$

at the "center" of the hadron, $r/r_0 << 1$, i.e., it returns to the stationary states of the "free" quark at small distances at which the quark, at the same time as the energy, possesses a definite absolute value of its momentum projection. The normalization condition for the radial functions $R(r)$ and $\chi(r)$ is defined by the integrals

$$\int_0^{r_0} |R|^2 r^2\, dr^2 \qquad \text{and} \qquad \int_0^{r_0} |\chi|^2\, dr$$

respectively.

Because of the complicated forms of the functions $N(r^2)$ and $D(r^2)$ an explicit solution of (39) is impossible and numerical investigations are needed. This problem is beyond the scope of this paper.

We now study an approximate equation which is obtained by means of decomposing the functions $N(r^2)$ and $D(r^2)$ in power series of (r/r_0). The result leads to the Schrödinger equation with the spatial anharmonic oscillator potential. We restrict ourselves to terms of order r^2/r_0^2 and distinguish two cases, where a dominating contribution comes either from the energy value (16) or from the potential field (12). Such a difference depends on the Euler number $\text{Euler}(M) = 2 - 2N$. Consider the first case and assume, as before, $\chi = 3$ and $N = 3$. Then equation (33) acquires the form

$$\Delta\Psi + \frac{2m}{\hbar^2}\left(\epsilon_1 - \frac{1}{2} m\omega_1^2 r^2\right)\Psi = 0 \tag{40}$$

where

$$\epsilon_1 = \frac{1}{11}(mc^2 - Ee^2), \qquad \omega_1^2 = \frac{1}{121}\frac{1}{r_0^2}\left(12c^2 - \frac{E}{m}e^2\right), \qquad e = 2.73.$$

Equation (40) means that the quark moves in the spatial oscillator field $U = \frac{1}{2}m\omega_1^2(x^2 + y^2 + z^2)$ and admits separating variables, which leads to three equations of the type of the linear oscillator. Therefore the energy levels are

$$\epsilon_1 = \hbar\omega_1(n_1 + n_2 + n_3 + \tfrac{3}{2}) \equiv \hbar\omega_1(n + \tfrac{3}{2}). \tag{41}$$

In this case, there are $(n + 1)(n + 2)/2$-fold degeneracies of nth level, which is equal to the number of combinations by which n may be represented in the form of the sum of three integer (including zero) positive numbers; that is, just $(n + 1)(n + 2)/2$. The wave function of the stationary state is

$$\Psi_{n_1 n_2 n_3} = \text{const}\cdot\exp\left(-\rho^2 \frac{r^2}{2}\right) H_{n_1}(\lambda x)H_{n_2}(\lambda y)H_{n_3}(\lambda z) \tag{42}$$

where $H_n(x)$ are the Hermitian polynomials and $\rho = \sqrt{m\omega_1/\hbar}$.

On changing the sign of the coordinate, the polynomials H_n are multiplied by $(-1)^n$; therefore the parity of the function (42) is $(-1)^{n_1+n_2+n_3} = (-1)^n$.

Making use of linear combinations of these functions with the given sum $n_1 + n_2 + n_3 = n$, one can form functions

$$\Psi_{nlm} = \text{const}\cdot e^{-\rho^2 r^2/2} r^n \theta_{lm}(\theta) e^{\pm im\varphi} \tag{43}$$

Here $\theta_{lm}(\theta)$ is given by expression (37), and $m = 0, 1, \ldots, l$, where

$$l = \begin{cases} 0, 2, \ldots, n & \text{for even } n \\ 1, 3, \ldots, n & \text{for odd } n \end{cases}$$

It is obvious that the latter is defined from a comparison between the parity $(-1)^n$ of the function (42) and the parity $(-1)^l$ of the function (43), which should be equal. In this manner, possible values of the orbital momenta corresponding to considerable energy levels are defined. It should be noted that in our case the energy levels of (40) are richer with respect to the usual spatial oscillator problem. Indeed, our true energy levels are given by

$$(E_n^1)_{1,2} = e^{-2}\left[mc^2 \pm \frac{c}{r_0}\hbar\left(n + \frac{3}{2}\right)\sqrt{11 + \frac{1}{4}\frac{m^{-2}}{c^2} r_0^{-2}\hbar^2\left(n + \frac{3}{2}\right)^2}\right.$$
$$\left. - \frac{\hbar^2}{2m} r_0^{-2}\hbar^2\left(n + \frac{3}{2}\right)^2\right] \tag{44}$$

in accordance with formula (41).

Let us consider the second case, where the main contribution arises from formula (12). Assuming again $\chi = 3$ and $N = 0$, we get

$$\Delta\Psi + \frac{2m}{\hbar^2}\left(\epsilon_2 - \frac{1}{2} m\omega_2^2 r^2\right)\Psi = 0 \tag{45}$$

where

$$\epsilon_2 = \frac{1}{24}\left(\frac{2E}{e^4} - mc^2\right), \qquad \omega_2^2 = \frac{1}{12 r_0^2}\left(\frac{c^2}{2} - \frac{3E}{e^4 m}\right)$$

The energy spectrum now acquires the form

$$(E_n^2)_{1,2} = e^4\left\{\frac{mc^2}{2} \pm \frac{2}{r_0}\hbar c\left(n+\frac{3}{2}\right)\sqrt{\left[9\hbar\left(n+\frac{3}{2}\right)\frac{1}{c^2r_0m}\right]^2 - 3} - 18\hbar^2 r_0^{-2}\frac{1}{m}\left(n+\frac{3}{2}\right)^2\right\} \tag{46}$$

For completeness we consider again the case where both formulas (12) and (16) give the same order of contribution to the quantities E' and $U'(r)$ in (33), that is, $\chi = 3$ and $N = 1$, and therefore

$$\epsilon_3 = E\left(1+\frac{25}{2}\frac{r^2}{r_0^2}\right), \qquad U_3' = \frac{1}{2}m\omega_3^2 r^2, \qquad \omega_3^2 = \frac{1}{r_0^2}\left(25\frac{E}{m} - \frac{71}{2}c^2\right)$$

For the given case, the energy spectrum yields

$$(E_n^3)_{1,2} = \frac{3}{2}mc^2 \pm \frac{\hbar c}{2}\left(n+\frac{3}{2}\right)\sqrt{2+\frac{625}{4}\frac{\hbar^2}{m^2r_0^2c^2}\left(n+\frac{3}{2}\right)^2} + \frac{25}{2}\frac{\hbar^2}{mr_0^2}\left(n+\frac{3}{2}\right)^2 \tag{47}$$

Assuming $r_0 = \hbar/m_pc$ and collecting these three cases, we have:

1. $(E_n^1)_1 = e^{-2}mc^2$,

$$(E_n^1)_2 = e^{-2}\left[mc^2 - \frac{m_p^2c^2}{m}\left(n+\frac{3}{2}\right)^2\right] \quad \text{for} \quad N = 3 \tag{48a}$$

2. $(E_n^2)_1 = \frac{1}{2}e^4mc^2$,

$$(E_n^2)_2 = e^4\left[\frac{1}{2}mc^2 - 36\frac{m_p^2c^2}{m}\left(n+\frac{3}{2}\right)^2\right] \quad \text{for} \quad N = 0 \tag{48b}$$

3. $(E_n^3)_1 = \frac{3}{2}mc^2$,

$$(E_n^3)_2 = \frac{3}{2}mc^2 + 25\frac{m_p^2c^2}{m}\left(n+\frac{3}{2}\right)^2 \quad \text{for} \quad N = 1 \tag{48c}$$

We see that fluctuation of the space-time metric around the hadron matter caused by the random string field yields a very varied spectrum of excited hadron states. Among them there exist states with enormously high energy levels, depending on the Euler number.

5. QUANTUM FIELD DESCRIPTION OF CONFINED QUARKS AND GLUONS

5.1. The Confinement Potential and Propagator of Confined Particles

The field concept description of the quark and gluon inside the hadron is carried out by the same principle as is applied to the construction of the usual theory of interacting particles, for example, photons and electrons. Notice that, in contrast to the familiar constituent quarks, there is no talk of the gluon as a "constituent." The concept of the gluon field is just needed to formulate a description of hadron matter by means of interacting quantized fields.

The fundamental quantity in the field theory is the particle propagator, or Green function, which reflects the causal connection of quantized fields propagating between points of space-time.

To define this quantity in our case, we recall the well-known fact that the form of the potential in the static limit is connected with the form of the particle propagator. For example, the Coulomb and Yukawa potentials are responsible for the existence of photon and scalar particle-transmitting quanta:

$$\frac{e}{4\pi r} = \frac{e}{(2\pi)^3}\int d^3p\, e^{i\mathbf{pr}}\,\frac{1}{\mathbf{p}^2} \tag{49}$$

$$\frac{g}{4\pi r}\,e^{-mr} = \frac{g}{(2\pi)^3}\int d^3p\, e^{i\mathbf{pr}}\,\frac{1}{m^2+\mathbf{p}^2} \tag{50}$$

Without loss of generality, we do not consider the tensor structure of the interacting confined particle propagator, and define its form in accordance with the general rules (49) and (50). For this purpose, we define the confinement potential, in our case due to the self-referential effect, the contributions to which arise from both the energy value (16) and the external potential (12). The full potential is exactly defined from the total energy (22a) of the particle when it is at rest. That is,

$$U_{\text{tot}}(r) = D(r^2) = \begin{cases} \left[\dfrac{g_1}{4\pi r_0}\left(1-\dfrac{r^2}{r_0^2}\right)^{1/2} + \dfrac{g_2}{4\pi r_0}\left(1-\dfrac{r^2}{r_0^2}\right)\right], & 0 \le r < r_0 \\ 0, & r_0 < r < \infty \end{cases} \tag{51}$$

where $g_1 = 4\pi r_0 mc^2\rho'$ and $g_2 = 2\pi r_0 mc^2\rho$.

In the general case, the quantities ρ', ρ, and r_0 depend on the external hadron momentum $\mathbf{Q}$,

$$\rho' = \exp\left\{\frac{\lambda^2}{\alpha_0'(1 - Q^2/c^2)}(1 - N)\right\},$$
$$\rho = (\rho')^4,$$
$$r_0 = \left[4\pi\alpha_0'\left(1 - \frac{Q^2}{c^2}\right)\right]^{1/2}$$

Thus we have the integral equation

$$\frac{g_1}{4\pi r_0}\left(1 - \frac{r^2}{r_0^2}\right)^{1/2} + \frac{g_2}{4\pi r_0}\left(1 - \frac{r^2}{r_0^2}\right)$$
$$= \frac{c_1}{(2\pi)^3}\int d^3p\, e^{i\mathbf{pr}}D_1(\mathbf{p}^2) + \frac{c_2}{(2\pi)^3}\int d^3p\, e^{i\mathbf{pr}}D_2(\mathbf{p}^2)$$

To solve this equation, we integrate it over the angular variables. The result reads

$$\frac{g_1}{4\pi r_0}\left(1 - \frac{r^2}{r_0^2}\right)^{1/2} + \frac{g_2}{4\pi r_0}\left(1 - \frac{r^2}{r_0^2}\right)$$
$$= \frac{1}{2\pi^2 r}\int_{r_0}^{\infty} dp\, p \sin pr\, [c_1D_1(p^2) + c_2D_2(p^2)] \tag{52}$$

This integral equation has the unique solution

$$D(\mathbf{p}^2) = g_1\frac{\pi r_0^2}{2}\frac{J_2(r_0\sqrt{\mathbf{p}^2})}{(r_0\sqrt{\mathbf{p}^2})^2} + \frac{g_2\sqrt{2\pi}r_0^2J_{5/2}(r_0\sqrt{\mathbf{p}^2})}{(r_0\sqrt{\mathbf{p}^2})^{5/2}} \tag{53}$$

Here we have chosen the constants c_1 and c_2 in such a way that equation (52) holds at the choice of the function (53), and $J_2(x)$ and $J_{5/2}(x)$ are the Bessel functions of the order of 2 and 5/2, respectively.

It is natural to generalize expression (53) for the relativistic case

$$D(p^2) = \frac{\pi}{2}r_0^2g_1(r_0\sqrt{-p^2})^{-2}J_2(r_0\sqrt{-p^2})$$
$$+ \sqrt{2\pi}r_0^2g_2(r_0\sqrt{-p^2})^{-5/2}J_{5/2}(r_0\sqrt{-p^2}) \tag{54}$$

and for the particle with mass M (i.e., a gluonlike particle with mass M)

$$D_M(p^2) = \frac{\pi}{2}r_0^2g_1(r_0\sqrt{M^2 - p^2})^{-2}J_2(r_0\sqrt{M^2 - p^2})$$
$$+ \sqrt{2\pi}r_0^2g_2(r_0\sqrt{M^2 - p^2})^{-5/2}J_{5/2}(r_0\sqrt{M^2 - p^2}) \tag{55}$$

where $p^2 = p_0^2 - \mathbf{p}^2$. The latter gives the oscillation potential

$$U_M = \begin{cases} \dfrac{g_1}{2\sqrt{2\pi}} \dfrac{1}{r_0^2\sqrt{M}} \left(1 - \dfrac{r^2}{r_0^2}\right)^{1/4} J_{1/2}(M\sqrt{r_0^2 - r^2}) & \\ \quad + \dfrac{g_2}{2\pi} \dfrac{1}{r_0^2 M} \left(1 - \dfrac{r^2}{r_0^2}\right)^{1/2} J_1(M\sqrt{r_0^2 - r^2}) & \text{for } 0 \le r < r_0 \\ 0 & \text{for } r_0 \ge r < \infty \end{cases} \tag{56}$$

If we assume that force-transmitting quanta giving the confinement potentials (51) and (56) are gluons and Z-boson-like objects (gluons with mass M), then expressions (54) and (55) may be written in the convenient form (inserting the tensor structure, which is formed by using the Lorentz and color indices)

$$D(p^2) \Rightarrow G_{\mu\nu}(p) = V_0(-p^2 r_0^2) G^0_{\mu\nu}(p) \tag{57}$$

$$D_m(p^2) \Rightarrow G^M_{\mu\nu}(p) = V_M(-p^2 r_0^2) G^{0M}_{\mu\nu}(p) \tag{58}$$

where

$$V_M(-p^2 r_0^2) = \tfrac{1}{2}\pi J_2(r_0\sqrt{M^2 - p^2}) + \sqrt{\pi/2}\, \rho'^3 (r_0\sqrt{M^2 - p^2})^{-1/2} J_{5/2}(r_0\sqrt{M^2 - p^2}) \tag{59}$$

$$V_0(-p^2 r_0^2) = V_M(-p^2 r_0^2)\,|_{M=0} \tag{60}$$

$G^0_{\mu\nu}(p^2)$ and $G^{0M}_{\mu\nu}(p^2)$ are the usual local propagators of gluons and massive gluons, respectively. We call the latter color Z-like bosons. We see that the propagators (57) and (58) possess some interesting properties:

1. They are entire analytic functions on the variable p^2.
2. Their order of growth is $\rho = 1/2$.
3. They decrease rapidly enough in the Euclidean directions $p^2 \Rightarrow -\infty$.
4. They have no poles in the complex plane p^2, so that force-transmitting quanta corresponding to these propagators are never observable.
5. Wave functions of these quanta are spread out over space-time,

$$G^{\text{nonloc}}_\mu(x) = \int d^4y\, K_0(x - y) G_\mu(y) \tag{61}$$

or

$$\tilde{Z}^{\text{nonloc}}_\mu(x) = \int d^4y\, K_m(x - y) \tilde{Z}_\mu(y) \tag{62}$$

where $K_0(x)$ and $K_m(x)$ are nonlocal generalized functions (Namsrai, 1986), the Fourier transforms of which are defined by $[V_0(r_0^2 p^2)]^{1/2}$ and

$[V_m(r_0^2p^2)]^{1/2}$, respectively. The local wave functions of gluons $G_\mu(x)$ and color Z-like bosons $\tilde{Z}_\mu(x)$ yield the usual propagators

$$\langle O|T\{G_\mu(x)G_\nu(y)\}|O\rangle = \frac{1}{i}\Delta^0_{\mu\nu}(x-y)$$

$$\langle O|T\{\tilde{Z}_\mu(x)\tilde{Z}_\nu(y)\}|O\rangle = \frac{1}{i}\Delta^M_{\mu\nu}(x-y) \tag{63}$$

by means of their T-product operators in x space.

Concerning the quark propagator, it should be constructed by the same procedure as just been defined for the gluon one. Thus, the confinement propagator for the quark takes the same form as (57) and (58):

$$\hat{S}_R(p) = V_m(-p^2r_0^2)\hat{S}^0(p) \tag{64}$$

where $\hat{S}^0(p)$ is the usual local propagator of the quark, which is used in QCD, and the form factor $V_m(-p^2r_0^2)$ is given by expression (59), where M should be changed to m, the quark mass. Thus, we see that the confinement potential due to the induced gravitation of "hadron" strings leads to a new type of "strong" interaction in which there exist two decreasing coupling constants $g_1 = 4\pi r_0 m_q c^2$ and $g_2 = 2\pi r_0 m_q^4 c^2/m^3$ at $N \Rightarrow \infty$ and $|Q| \Rightarrow c$ [Euler(M) = $2 - 2N$, $\mathbf{Q}$ is the external momentum of the hadron]. The corresponding force-transmitting quanta become nonlocal and unobservable, and their propagators are entire analytic functions. The quantum field theory for such interactions is constructed by means of the Efimov (1985) method in which there are no ultraviolet divergences.

5.2. The Lagrangian Function of the Confined Particle and Its Quantization Procedure

5.2.1. Definition of the Corresponding Rule

Without loss of generality, we consider a one-component confined scalar particle with mass m and the propagator

$$V_m(-p^2r_0^2)\Delta(p) \tag{65}$$

where $\Delta(p)$ is the usual propagator of the scalar particle in the momentum space. The question arises of how to find the Lagrangian $\mathcal{L}$ of such a field by knowing its propagator.

We act as follows. As is well known, in the traditional approach to the description of the quantum field, the initial object of construction is a given Lagrangian by which the propagators and vertices are formed. Let us consider the most general Lagrangian

$$\mathcal{L}(x) = \psi_i^*(x)V_{ij}\psi_j(x) + \tfrac{1}{2}\varphi_i(x)W_{ij}\varphi_j(x) + \mathcal{L}_I(\psi^*, \psi, \varphi)$$

The ψ_i and φ_i denote sets of complex and real fields that may be scalar, spinor, vector, tensor, etc., fields. The index i stands for any spinor, Lorentz, isospin, color, etc. V and W are matrix operators that may contain derivatives, and whose Fourier transform must have an inverse. The interaction Lagrangian is allowed to be nonlocal, i.e., it depends not only on fields at the point x, but also on fields at other space-time points x', x'' .... The coefficients in the polynomial expansion may be functions of x.

The explicit form of a general term in $\mathcal{L}_I(x)$ is

$$\int d^4x_1 \cdots d^4x_n \cdots K_{i_1 i_2} \cdots (x, x_1, x_2, \ldots)\psi_{i_1}^*(x_1) \cdots \psi_{i_m}(x_m) \cdots \varphi_{i_n}(x_n) \cdots$$

The K may contain any number of differential operators working on the various fields. Roughly speaking, the propagators are defined to be minus the inverse of the Fourier transforms of V and W, and the vertices as the Fourier transforms of the coefficients K in $\mathcal{L}_I$ [for details, see t'Hooft and Veltman (1973)].

For example, the propagators are minus the inverse of the operator found in the quadratic term:

$$\mathcal{L} = \frac{1}{2}\varphi(x)[\Box - m^2]\varphi(x) \Rightarrow (m^2 - p^2 - i\epsilon)^{-1}$$

$$\mathcal{L} = \bar{\psi}(i\hat{\partial} - m)\psi \Rightarrow \frac{m - \hat{p}}{m^2 - p^2 - i\epsilon}$$

In contrast to the usual theory, in our scheme, the starting point is provided by the propagators (57) and (58) of entire analytic functions, and therefore we attempt to construct Lagrangian corresponding to these by using the above-mentioned rule. Thus, for the scalar particle case one gets

$$\frac{V_m(p^2r_0^2)}{m^2 - p^2 - i\epsilon} \Rightarrow \mathcal{L}_f = \frac{1}{2}\phi(x)E(\Box)\phi(x) \tag{66}$$

where

$$\phi(x) = K_m(\Box)\varphi(x), \qquad E(\Box) = \frac{\Box - m^2}{V_m(r_0^2\Box)} \tag{67}$$

The operator $K_m(\Box)$ in (67) is nonlocal and may be represented in the form

$$K_m(\Box) = \sum_{n=0}^{\infty} \frac{c_n}{(2n)!}(r_0^2\Box)^n$$

The generalized function $K_m(x - x') = K(\Box)\delta^{(4)}(x - x')$ in (61) and (62) belongs to one of the spaces of the nonlocal generalized functions considered in Efimov (1977, 1985). In the general case, when there exists some interaction term $gU(\phi(x))$ describing the self-interaction of the confined scalar field $\phi(x)$, we have

$$\mathcal{L} = \tfrac{1}{2}\phi(x)E(\Box)\phi(x) + gU(\phi(x)) \tag{68}$$

Our next problem is to quantize this system. We use the Efimov (1977, 1985) method. Formally, the equation of motion reads

$$E(\Box)\phi(x) = -gU'(\phi(x)) \tag{69}$$

for the interacting field and

$$E(\Box)\phi(x) = \frac{\Box - m^2}{V_m(r_0^2\Box)}\phi(x) = 0 \tag{70}$$

for the free field.

For convenience, in what follows we omit the index m on $V_m(-l^2\Box)$. The problem is how to understand these equations, how to study and solve them, and how to perform the quantization of the field $\phi(x)$. We introduce a certain regularization into the classical Lagrangian in a way that permits us to carry out the usual canonical quantization. This quantization leads to the appearance of additional ghost states with indefinite metrics. The ghost states disappear when the regularization is removed, but a trace remains, namely the propagator of the scalar particle becomes nonlocal according to (65). Thus, instead of (69) or (70), we consider the regularized equation

$$E^\delta(\Box)\phi^\delta(x) = -gU'(\phi^\delta(x)) \tag{71}$$

or, in the case $g = 0$,

$$E^\delta(\Box)\phi^\delta(x) = 0 \tag{72}$$

Here δ is a parameter of the regularization such that

$$\lim_{\delta\Rightarrow 0} E^\delta(\Box) = E(\Box) = \frac{\Box - m^2}{V(r_0^2\Box)} \tag{73}$$

Instead of the Lagrangian (68), we obtain

$$\mathcal{L}(x) = \tfrac{1}{2}\phi^\delta(x)E^\delta(\Box)\phi^\delta(x) + gU(\phi^\delta(x)) \tag{74}$$

The regularization is chosen in such a way that the function

$$E^\delta(k^2) = \frac{k^2 - m^2}{V(-r_0^2k^2)}$$

has zeros in some set of points:

$$E^{\delta}(k^2) \sim (k^2 - m^2) \prod_{j=1}^{\infty} [k^2 - m_j^2(\delta)] \tag{75}$$

so that $m_j^2(\delta) > 0$ $(j = 1, 2, \ldots)$ and $m_j^2(\delta) \Rightarrow \infty$ when $\delta \Rightarrow 0$.

Then the field $\phi^{\delta}(x)(\delta > 0)$ can be quantized by methods using indefinite metrics (Pais, and Uhlenbeck, 1950; Nady, 1966). The Hamiltonian H_0^{δ} and the vector space of states H^{δ} with indefinite metrics can be constructed for the free system when $\delta > 0$. Further, the S^{δ}-matrix, and also different operators such as the current

$$j^{\delta}(x) = i[\delta S^{\delta}/\delta\Phi^{\delta}(x)]S^{\delta^+}$$

the Green functions

$$G^{\delta}(x - y) = \langle 0 | T[\Phi^{\delta}(x)\Phi^{\delta}(y)] | 0 \rangle$$

and the Wightman function

$$W^{\delta}(x_1, \ldots, x_n) = \langle 0 | \Phi^{\delta}(x_1) \cdots \Phi^{\delta}(x_n) | 0 \rangle$$

can be found for the confined interacting system.

By definition, we consider that when $\delta \Rightarrow 0$ the limits of all these physical quantities are the quantum field solution of the initial system (68). The problem indicates such a regularization procedure, which provides the existence of the limits of all operators and matrix elements for all physical quantities at $\delta \Rightarrow 0$.

This means that we have to obtain a self-consistent theory in the limit $\delta \Rightarrow 0$.

5.2.2. Regularization Procedure

Let us consider the regularized function

$$\begin{aligned} V^{\delta}(-z\lambda) &= \sum_{n=0}^{\infty} \left\{ \frac{u_n(z - 1)^n}{\prod_{j=1}^{n+2}[1 - \delta/j^{\sigma}(z - 1)]} \right\} \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n u_n(z - 1)^n [\Gamma(n + 3)]^{\sigma}}{\delta^{n+2} \prod_{j=1}^{n+2} [z - \mu_j(\delta)]} \end{aligned} \tag{76}$$

[ρ is the growth order of $V(-r_0^2k^2)$] instead of the function

$$V(-r_0^2k^2) = \sum_{n=0}^{\infty} v_n(k^2 - m^2)^n r_0^{2n}$$

where we have denoted

$$z = k^2/m^2, \qquad \lambda = m^2 r_0^2, \qquad u_n = v_n \lambda^n, \qquad \sigma < 1/\rho \leq 2,$$

$$\mu_j(\delta) = 1 + j^\sigma/\sigma$$

From the form of the function

$$E^\delta(z) = (z - 1)/V^\delta(-z\lambda) \tag{77}$$

it follows that the function

$$\frac{1}{E^\delta(z)} = \frac{V^\delta(-z\lambda)}{z - 1} \tag{78}$$

is a meromorphic analytic function in the complex z-plane and has simple poles at the points

$$z = \mu_j(\delta) = 1 + \frac{j^\sigma}{\delta} \qquad (j = 1, 2, \ldots)$$

and decreases as [except the ray (z: arg $z = 0$)]

$$1/E^\delta(z) \approx |z|^{-3} \sum_{n=0}^{\infty} (-1)^n \delta^{-n-2} u_n [\Gamma(n + 3)]^\sigma$$

when $|z| \Rightarrow \infty$. Here the series converges (Efimov, 1977). For the meromorphic function (78) the following representation is valid:

$$1/E^\delta(z) = \sum_{j=0}^{\infty} \left\{ \frac{(-1)^j A_j(\delta)}{z - \mu_j(\delta)} \right\} \tag{79}$$

where

$$A_j(\delta) = \sum_{n=\max(0,j-2)} u_n c_{n_j} (j^\sigma/\delta)^n \tag{80}$$

$$c_{n_j} = \frac{[\Gamma(n + 3)]^\sigma}{\prod_{i=0}^{j-1}(j^\sigma - i^\sigma)\prod_{k=j+1}^{n+2}(k^\sigma - j^\sigma)} \tag{81}$$

The numbers $A_j(\delta)$ satisfy the condition

$$\sum_{n=0}^{\infty} (-1)^j A_j(\delta)[\mu_j(\delta)]^N = 0 \qquad \text{for} \quad N = 0, 1, 2 \tag{82}$$

The inequality ('T Hooft and Veltman, 1973)

$$A_j(\delta) \leq \frac{\text{const} \cdot \exp\{B(j^\sigma/\delta)^{1/2}\}}{\Gamma((2 - \sigma)j)}$$

is valid for the number $A_j(\delta)$ at large j, where B is some number and $(1/2)\sigma < 1$.

Let us now consider the function

$$D^\delta(k^2) = -[E^\delta(k^2)]^{-1}$$

which has the following properties.

(1) It is a meromorphic analytic function in the complex k^2-plane and has simple poles at the points

$$k^2 = m_j^2(\delta) = m^2\mu_j(\delta) = m^2\left(1 + \frac{j^\sigma}{\delta}\right) \qquad (j = 1, 2, \ldots)$$

(2) The residues of $D^\delta(k^2)$ at these points are given by

$$\text{Res}\, D^\delta(k^2) = (-1)^{j+1}A_j(\delta), \qquad k^2 = m_j^2(\delta)$$

(3) When $|k^2| \Rightarrow \infty$ in all k^2 planes [except the ray (m^2, ∞)]

$$D^\delta(k^2) = \frac{V^\delta(-k^2r_0^2)}{(m^2 - k^2)} = O\left(\frac{1}{|k^2|^3}\right)$$

(4) The function $D^\delta(k^2)$ may have zeros at the points

$$k^2 = a_r \qquad (r = 0, 1, 2, \ldots)$$

(5)

$$\lim_{\delta \Rightarrow 0} D^\delta(k^2) = \frac{V(-k^2r_0^2)}{(m^2 - k^2)}$$

i.e., it becomes an entire analytic function in accordance with (59).

5.2.3. Quantization of the Regularized Equation

Let us consider the classical system described by the Lagrangian density $\mathcal{L}^\delta(x)$, (74), where the regularized operator $E^\delta(\Box)$ satisfies the properties enumerated above. According to the principle of stationary action, the wave equation for the system described by (74) has the form (71). It is a differential equation of infinite order, i.e., it is an integral equation.

In order to solve the Cauchy problem we have to know the values of the function $\phi^\delta(x)$ at all its derivatives at the initial time.

We analyze the solution of this equation following the scheme proposed by Pais and Uhlenbeck (1950). Let us introduce the system of fields

$$\phi_j^\delta = [A_j(\delta)]^{1/2}\{\tilde{E}^\delta(\Box)\}\phi^\delta(x) \tag{83}$$

where the coefficients $A_j(\delta)$ are introduced by formula (80),

$$m_j^2(\delta) = m^2(1 + j^\sigma/\delta) \qquad (j = 0, 1, 2, \ldots) \tag{84a}$$

and

$$\tilde{E}_j^\delta(\Box) = \frac{E^\delta(\Box)}{[\Box - m_j^2(\delta)]} \tag{84b}$$

According to definition (83), the fields $\phi_j^\delta(x)$ are not independent for different j and they satisfy the relations

$$[A_j(\delta)]^{1/2}\{\tilde{E}_j^\delta(\Box)\}\phi_i^\delta(x) = [A_i(\delta)]^{1/2}\{\tilde{E}_i^\delta(\Box)\}\phi_j^\delta(x) \tag{85}$$

The field $\phi^\delta(x)$ can be expressed as

$$\phi^\delta(x) = \sum_{j=0}^{\infty} (-1)^j [A_j(\delta)]^{1/2}\phi_j^\delta(x) \tag{86}$$

In fact, on the one hand, the chain of equalities

$$\phi^\delta = \sum_{j=0}^{\infty} (-1)^j A_j(\delta)\tilde{E}_j^\delta(\Box)\phi^\delta(x) = \left[\frac{1}{E^\delta(\Box)}\right]E^\delta(\Box)\phi^\delta(x) = \phi^\delta(x)$$

is valid. On the other hand, using (85), it is possible to obtain

$$\begin{aligned}
\phi_j^\delta(x) &= [A_j(\delta)]^{1/2}\tilde{E}_j^\delta(\Box) \sum_{i=0}^{\infty} (-1)^i [A_i(\delta)]^{1/2}\phi_i^\delta(x) \\
&= \sum_{i=0}^{\infty} (-1)^i [A_i(\delta)]^{1/2}[A_i(\delta)]^{1/2}\tilde{E}_i^\delta(\Box)\phi_j^\delta(x) \\
&= \sum_{i=0}^{\infty} (-1)^i \left\{\frac{A_i(\delta)}{\Box - m_i^2(\delta)}\right\} E^\delta(\Box)\phi_j^\delta(x) = \phi_j^\delta(x)
\end{aligned}$$

On the basis of the correlations (83), (85), and (86), the Lagrangian density can be expressed in terms of the fields $\phi_j^\delta(x)$

$$\begin{aligned}
\mathcal{L}^\delta(x) &= \frac{1}{2}\sum_{j=0}^{\infty} (-1)^j \phi_j^\delta(x)[\Box - m_j^2(\delta)]\phi_j^\delta(x) \\
&\quad + gU\left\{\sum_{j=0}^{\infty} (-1)^j [A_j(\delta)]^{1/2}\phi_j^\delta(x)\right\}
\end{aligned} \tag{87}$$

Assuming the fields $\Phi_j^\delta(x)$ to be independent variables, and making use of the principle of stationary action, we obtain from (87) the infinite system of equations

$$[\Box - m_j^2(\delta)]\phi_j^\delta(x) = -g[A_j(\delta)]^{1/2}U'\left\{\sum_{i=0}^{\infty}(-1)^i[A_i(\delta)]^{1/2}\phi_i^\delta(x)\right\} \quad (88)$$

The system of equations obtained here is equivalent to (71). Indeed, first, the relation (85) results directly from the system of equations (88); second, (71) is fulfilled if $\phi^\delta(x)$ is connected with the fields $\phi_j^\delta(x)$ by the relation (86), and making use of (86), (85), and (88), it is easy to verify that

$$\begin{aligned}
E^\delta(\Box)\phi^\delta(x) &= E^\delta(\Box)\sum_{j=0}^{\infty}(-1)^j[A_j(\delta)]^{1/2}\phi_j^\delta(x)\\
&= \sum_{j=0}^{\infty}(-1)^j[A_j(\delta)]^{1/2}\left\{\frac{\Box - m_i^2(\delta)}{[A_i(\delta)]^{1/2}}\right\}[A_i(\delta)]^{1/2}\tilde{E}_i^\delta\phi_j^\delta(x)\\
&= \frac{\Box - m_i^2(\delta)}{[A_i(\delta)]^{1/2}}\sum_{j=0}^{\infty}(-1)^j A_j(\delta)\tilde{E}_j^\delta\phi_i^\delta(x)\\
&= [\Box - m_i^2(\delta)][A_i(\delta)]^{-1/2}\phi_i^\delta(x)\\
&= -gU'(\phi^\delta(x))
\end{aligned}$$

where $i = 0, 1, 2, \ldots$.

Thus the Lagrangian (87) and the system of equations (88) are completely equivalent to the Lagrangian (74) and (71). Therefore, one can assume that our initial system (74) is described by the Lagrangian (87), where the fields $\phi_j^\delta(x)$ are independent and satisfy the equation of motion (88). The method used here is well known in the theory of differential equations. It is used in general when a differential equation of the highest order is replaced by a system of differential equations of the first order. All the above arguments concern classical field theory. The quantization of the system of classical fields $\{\phi_j^\delta(x)\}$ can be performed according to the canonical procedure of quantization. Let us introduce a momentum field conjugate to $\phi_j^\delta(\mathbf{x}, 0)$,

$$\Pi_j^\delta(\mathbf{x}, 0) = [\delta/\delta\dot{\phi}_j^\delta(\mathbf{x}, 0)]\int d^3y\, \mathcal{L}^\delta(\mathbf{y}, 0) = (-1)^j\dot{\phi}_j^\delta(\mathbf{x}, 0) \quad (89)$$

The dot denotes the differential with respect to time:

$$\dot{\phi}_j^\delta(\mathbf{x}, 0) = \frac{\partial}{\partial t}\phi_j^\delta(\mathbf{x}, t)|_{t=0}$$

We treat ϕ_j^δ and Π_j^δ as operators with the commutation relations

$$\begin{aligned}
[\phi_j^\delta(\mathbf{x}, 0), \phi_i^\delta(\mathbf{y}, 0)]_- &= [\Pi_j^\delta(\mathbf{x}, 0), \Pi_i^\delta(\mathbf{y}, 0)]_- = 0\\
[\phi_j^\delta(\mathbf{x}, 0), \Pi_i^\delta(\mathbf{y}, 0)]_- &= i\delta_{ij}\delta(\mathbf{x} - \mathbf{y}) \quad (90)
\end{aligned}$$

or

$$[\phi_j^\delta(\mathbf{x}, 0), \dot{\phi}_i^\delta(\mathbf{y}, 0)]_- = i(-1)^j\delta_{ij}\delta(\mathbf{x} - \mathbf{y}) \tag{91}$$

It is seen that the indefinite metric is to be employed to quantize our system in a regular manner (Nady, 1966). As we are unable to solve the system of equations (88) exactly, our problem is to construct the perturbation series for the S-matrix and we perform the quantization of the noninteracting system of fields $\{\phi_j^\delta(x)\}$, i.e., the case when $g = 0$. Instead of (88) we have

$$[\Box - m_j^2(\delta)]\phi_j^\delta(x) = 0 \qquad (j = 0, 1, 2, \ldots) \tag{92}$$

The solution of these equations can be written in the form

$$\phi_j^\delta(x) = (2\pi)^{-3/2}\int d^3k\, [2\omega_{j\mathbf{k}}^\delta]^{-1/2}(d_{j\mathbf{k}}e^{-i\mathbf{k}\mathbf{x}} + d_{j\mathbf{k}}^+e^{i\mathbf{k}\mathbf{x}}) \tag{93}$$

where

$$\omega_{j\mathbf{k}}^\delta = [\mathbf{k}^2 + m_j^2(\delta)]^{1/2} = [\mathbf{k}^2 + m^2(1 + j^\sigma/\delta)]^{1/2}$$

From the quantization conditions (90) and (91), the operators $d_{j\mathbf{k}}$ and $d_{j\mathbf{k}}^+$ satisfy

$$[d_{j\mathbf{k}}, d_{j'\mathbf{k}'}] = [d_{j\mathbf{k}}^+, d_{j'\mathbf{k}'}^+] = 0$$

$$[d_{j\mathbf{k}}, d_{j'\mathbf{k}'}^+] = (-1)^j\delta_{jj'}\delta(\mathbf{k} - \mathbf{k}') \tag{94}$$

The Hamiltonian of the noninteracting system can easily be obtained. Let us write it in the normal form

$$H_0^\delta = \sum_{j=0}^{\infty}(-1)^j\int d^3k\, \omega_{j\mathbf{k}}^\delta d_{j\mathbf{k}}^+ d_{j\mathbf{k}} \tag{95}$$

The system under consideration consists of quanta with the following mass spectra:

$$m_j^2(\delta) = \begin{cases} m^2, & j = 0 \\ m^2(1 + j^\sigma/\delta), & j = 1, 2, \ldots \end{cases} \tag{96}$$

Let us define

$$d_{0\mathbf{k}} = a_\mathbf{k} \qquad \text{and} \qquad d_{0,\mathbf{k}}^+ = a_\mathbf{k}^+ \tag{97}$$

When $\delta \Rightarrow 0$, the masses of quanta with $j = 1, 2, \ldots$ go to infinity, according to (96). These quanta are called ghost states or ghosts. The quanta with $j = 0$ have finite mass m. We call then normal particles or scalar particles with mass m. The space of states H^δ is a vector space with indefinite metric. It consists of:

(1) A vacuum state $|0\rangle$ that is unique, defined by the conditions

$$d_{j\mathbf{k}}|0\rangle = 0 \qquad \text{and normalized by} \qquad \langle 0|0\rangle = 1$$

(2) One-particle states $|j, \mathbf{k}\rangle = d_{j\mathbf{k}}^{+}|0\rangle$, which are normalized by

$$\langle j, \mathbf{k}|j', \mathbf{k}'\rangle = (-1)^{j}\delta_{jj'}\delta(\mathbf{k} - \mathbf{k}')$$

and are eigenstates of the Hamiltonian H_0^δ,

$$H_0^\delta|j, \mathbf{k}\rangle = \omega_{j\mathbf{k}}^\delta|j, \mathbf{k}\rangle$$

(3) Many-particle states. If there are n particles with momenta $\mathbf{k}_1, \ldots, \mathbf{k}_n$ and among them there are $\nu_1, \nu_2, \ldots, \nu_\alpha$ $(n = \nu_1 + \nu_2 + \cdots + \nu_\alpha)$ identical particles, i.e., with the same index j, then the following is given:

$$|n\rangle = |j_1, \mathbf{k}_1, \ldots, j_n, \mathbf{k}_n\rangle = (\nu_1! \cdots \nu_\alpha!)^{-1/2} d_{j_1\mathbf{k}_1}^{+} \cdots d_{j_n\mathbf{k}_n}^{+}|0\rangle$$

These states are also eigenstates of H_0^δ:

$$H_0^\delta|n\rangle = (\omega_{j_1\mathbf{k}_1}^\delta + \cdots + \omega_{j_n\mathbf{k}_n}^\delta)|0\rangle$$

All these states generate a complex system of eigenstates in the vector space H_0^δ, i.e.,

$$(*)_{\text{Def}} = |0\rangle\langle 0| + \sum_{n=1}^{\infty} \sum_{j_1,\ldots,j_n=0}^{\infty} (-1)^{j_1+\cdots+j_n} \int d^3k_1 \cdots \int d^3k_n\, |n\rangle\langle n| = 1$$

What happens with the space H^δ when $\delta \Rightarrow 0$? At $\delta \Rightarrow 0$, the masses of all ghosts increase according to (96). Therefore, if any physical state is characterized by a definite value of energy, then in the limit $\delta \Rightarrow 0$ no physical states with arbitrary but finite energy can consist of ghost quanta. In this sense we have

$$\lim_{\delta \Rightarrow 0} H^\delta = H \tag{98}$$

where H is the Hilbert space which contains (1) a vacuum state $|0\rangle$, $a_{\mathbf{k}}|0\rangle = 0$, and (2) single- and many-particle states

$$|n\rangle = |\mathbf{k}_1, \ldots, \mathbf{k}_n\rangle = (n!)^{-1/2} a_{\mathbf{k}_1}^{+} \cdots a_{\mathbf{k}_n}^{+}|0\rangle$$

All these states generate the complete system in H:

$$(*)_{\text{Def}} = |0\rangle\langle 0| + \sum_{n=1}^{\infty} \int d^3k_1 \cdots \int d^3k_n\, |\mathbf{k}_1, \ldots, \mathbf{k}_n\rangle\langle \mathbf{k}_1, \ldots, \mathbf{k}_n| = 1$$

5.2.4. Green Functions of the Field $\Phi^\delta(x)$

First let us consider the commutator

$$\Delta^\delta(x - y) = [\Phi^\delta(x), \Phi^\delta(y)]_{-} \tag{99}$$

Substituting the representations (86) and (99) and using (93), we obtain

$$\Delta^{\delta}(x) = \sum_{j=0}^{\infty} (-1)^j A_j(\delta)\Delta_j(x) \tag{100}$$

Calculation of the explicit form of the different two-point Green functions is carried out in many textbooks of field theory (see, for example, Bogolubov and Shirkov (1980). We use here their results. Thus one gets

$$\begin{aligned} \Delta_j^{\delta}(x) &= (2\pi)^{-3} \int d^4k\, \epsilon(k_0)\delta(k^2 - m_j^2(\delta))e^{-ikx} \\ &= \frac{1}{2\pi i}\epsilon(x_0)\delta(x^2) - \frac{m_j(\delta)}{4\pi i}(x^2)^{-1/2}\theta(x^2)J_1(m_j(\delta)(x^2)^{1/2}) \end{aligned} \tag{101}$$

where

$$\epsilon(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases} \qquad \theta(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

Because the series (100) converges absolutely, we have

$$\Delta^{\delta}(x) = 0 \qquad \text{when} \quad x^2 < 0 \tag{102}$$

Thus the operator $\Phi^{\delta}(x)$ satisfies the local commutation relations.

Let us now introduce the functions $\Delta^{\delta}_{\pm}(x)$ according to

$$\Delta^{\delta}_{(-)}(x - y) = \Delta^{\delta}_{(+)}(y - x) = \langle 0|\Phi^{\delta}(x)\Phi^{\delta}(y)|0\rangle$$

We have

$$\Delta^{\delta}_{\pm}(x) = \sum_{j=0}^{\infty} (-1)^j A_j(\delta)\Delta^{\delta}_{j\pm}(x) \tag{103}$$

where

$$\Delta^{\delta}_{j\pm}(x) = (2\pi)^{-3}\int d^4k\, \theta(\mp k_0)\delta(k^2 - m_j^2(\delta))e^{-ikx}$$

For $x^2 \Rightarrow 0$ one obtains (Bogolubov and Shirkov, 1980)

$$\begin{aligned} \Delta^{\delta}_{j\pm}(x) = &-\frac{i}{4\pi}\epsilon(\mp x_0)\theta(x^2) - \frac{1}{4\pi^2 x^2} + \frac{m_j^2}{16\pi^2}\ln\left(\frac{m_j^2|x^2|}{4}\right) + \left(\frac{im_j^2}{16\pi}\right) \\ &\times \epsilon(\mp x_0)\theta(x^2) + O(x^2 \ln x^2) \end{aligned}$$

Substituting this expansion into (103), we obtain

$$\Delta^{\delta}_{(\mp)}(x) = \frac{m^2}{16\pi^2}\sum_{j=0}^{\infty}(-1)^j A_j(\delta)\mu_j \ln \mu_j + O(x^2 \ln x^2), \qquad \mu_j(\delta) = 1 + \frac{j^\sigma}{\delta}$$

Here we have used the correlations (82); hence it appears that the function $\Delta^{\delta}_{(\pm)}(x)$ is finite at $x = 0$ and that

$$\Delta^{\delta}_{(\pm)}(0) = \frac{m^2}{16\pi^2}\sum_{j=0}^{\infty}(-1)^j A_j(\delta)\mu_j \ln \mu_j < \infty \tag{104}$$

This means that the operator $\Phi^\delta(x)$ is well defined because of $\langle 0 | \Phi^\delta(x)\Phi^\delta(x) | 0\rangle = \Delta^{\delta}_{(-)}(0) < \infty$.

Let us consider the causal Green function

$$\Delta^{\delta}_{c}(x-y) = \langle 0 | T[\Phi^\delta(x)\Phi^\delta(y)] | 0\rangle = \sum_{j=0}^{\infty}(-1)^j A_j(\delta)\Delta^{\delta}_{jc}(x-y) \tag{105}$$

where

$$\Delta^{\delta}_{jc}(x-y) = -i(2\pi)^{-4}\int d^4k\, e^{-ikx}[m_j^2(\delta) - k^2 - i\epsilon]^{-1}$$

Otherwise

$$\Delta^{\delta}_{c}(x) = -i(2\pi)^{-4}\int d^4k\, \tilde{\Delta}^{\delta}_{c}(k^2)e^{-ikx} \tag{106}$$

where

$$\begin{aligned}\tilde{\Delta}^{\delta}_{c}(k^2) &= \sum_{j=0}^{\infty}(-1)^j A_j(\delta)[m_j^2(\delta) - k^2 - i\epsilon]^{-1} \\ &= [m^2 - k^2 - i\epsilon]^{-1}\sum_{n=0}^{\infty}\frac{v_n r_0^{2n}(k^2 - m^2)^n}{\prod_{j=1}^{n+2}[1 - j^{-\sigma}m^{-2}\delta(m^2 - k^2 + i\epsilon)]}\end{aligned} \tag{107}$$

The function $\tilde{\Delta}^{\delta}_{c}(k^2)$ is analytic in the complex k^2 plane for Im $k^2 \geq 0$ and has simple poles at the point $k^2 = m_j^2(\delta) - i\epsilon$ ($j = 1, 2, \ldots$). The retarded and advanced Green functions can be defined in the following way:

$$\begin{aligned}\Delta^{\delta}_{\text{ret}}(x) &= \theta(x_0)\Delta^\delta(x) = \Delta^{\delta}_{c}(x) + \Delta^{\delta}_{(+)}(x) \\ \Delta^{\delta}_{\text{adv}}(x) &= -\theta(-x_0)\Delta^\delta(x) = \Delta^{\delta}_{c}(x) - \Delta^{\delta}_{(-)}(x)\end{aligned} \tag{108}$$

They satisfy the conditions

$$\Delta^{\delta}_{\text{ret}}(x) = 0 \qquad \text{for} \quad \begin{cases} x^2 < 0 \\ x^2 > 0, \quad x_0 < 0. \end{cases}$$

$$\Delta^{\delta}_{\text{adv}} = 0 \qquad \text{for} \quad \begin{cases} x^2 < 0 \\ x^2 > 0, \quad x_0 > 0 \end{cases} \tag{109}$$

Thus we can see that all Green functions satisfy all requirements of the local quantum field theory. This means that the field $\Phi^{\delta}(x)$ is local. The following usual correlations are valid for the regularized Green $\Delta_c^{\delta}(x)$ and $\Delta_{(\pm)}^{\delta}(x)$ functions:

$$\Delta_c^{\delta}(x) = \theta(x_0)\Delta_{(-)}^{\delta}(x) + \theta(-x_0)\Delta_{(+)}^{\delta}(x)$$
$$\Delta_c^{\delta *}(x) = \theta(x_0)\Delta_{(+)}^{\delta}(x) + \theta(-x_0)\Delta_{(-)}^{\delta}(x) \tag{110}$$

5.2.5. The Interacting System Before Removal of the Regularization

The interacting confined system is described by the Lagrangian (87). The total Hamiltonian of this system has the form

$$H^{\delta} = H_0^{\delta} + H_{\text{in}}^{\delta} \tag{111}$$

where H_0^{δ} is given by the relation (95) and

$$H_{\text{in}}^{\delta} = -g \int d^3x :U(\Phi^{\delta}(\mathbf{x}, 0)): \tag{112}$$

In the interaction picture, we have

$$H_{\text{in}}^{\delta}(t) = e^{-itH_0^{\delta}} H_{\text{in}}^{\delta} e^{itH_0^{\delta}} = -g \int d^3x :U(\Phi^{\delta}(\mathbf{x}, t)): \tag{113}$$

Although the operator $\Phi^{\delta}(x)$ is well defined, as we showed in the previous section, the Hamiltonian $H_{\text{in}}^{\delta}(t)$ is not defined, since the theory is translationally invariant and thus there are difficulties connected with the Haag theorem (Wightman, 1964). It is necessary to introduce the operation of "switching on" and "switching off" the interaction g:

$$g \Rightarrow g\left(\frac{\mathbf{x}}{L}, \frac{t}{L}\right) = g\left(\frac{x}{L}\right)$$

Such a regularization simultaneously takes into account first that our system is situated in a box, violating Euclidean invariance, and second that the interaction is adiabatically switched on and switched off at infinity, i.e., when $t \Rightarrow \pm\infty$. The large parameter L defines the intensity of switching on the interaction. The function $g(x)$ satisfies the conditions:

(1) $0 \leq g(x) < g$
(2) $g(0) = g$
(3) $[d^n g(x)]_{x=0} = 0$ for $n = 1, 2, 3, 4$, where $d^n = \partial_{\mu_1} \cdots \partial_{\mu_n}$
(4) $\int d^4x\, g(x) < \infty$
(5) $g(x) \in Z_{\alpha}$ for $\alpha < 2\rho/(2\rho - 1) = \infty$

where $\rho = 1/2$ is the order of growth of the nonlocal form factors (59) and (66).

Removal of the regularization corresponds to the limit $L \Rightarrow \infty$, at which

$$\lim_{L\Rightarrow\infty} g(x/L) \Rightarrow g(0) = g \tag{114}$$

Thus the Hamiltonian (113) in the interaction picture takes the form

$$H_{\rm in}^{(\delta,L)}(t) = -\int d^3x\, g(\mathbf{x}/L, t/L)\, :U(\Phi^\delta(\mathbf{x}, t)): \tag{115}$$

We recall that when δ is positive, then the Hamiltonian (115) is well defined, so that it is not necessary to introduce any ultraviolet cutoff. The regularized S-matrix can be written in the standard form

$$\begin{aligned} S^{\delta,L} &= T\exp\left\{-i\int_{-\infty}^{\infty} dt\, H_{\rm in}^{(\delta,L)}(t)\right\} \\ &= T\exp\left\{i\int d^4x\, g(x/L)\, :U(\Phi^\delta(x,t)):\right\} \end{aligned} \tag{116}$$

Here T has the meaning of a strict ordering operator of the quantized fields $\Phi^\delta(x)$ with respect to time. The chronological pairing of the Green function of the operators $\Phi^\delta(x)$ is given by formulas (105)–(107). Since the operators $\Phi^\delta(x)$ and $H_{\rm in}^{(\delta,L)}$ are well defined and local, then the $S^{\delta,L}$-matrix on the vector space H^δ is unitary and microcausal, i.e.,

$$\begin{aligned} S^{\delta,L}(S^{\delta,L})^+ &= S^{\delta,L} \otimes^\delta (S^{\delta,L})^+ = 1 \\ \frac{\delta}{\delta g(x)}\left\{\frac{\delta S^{\delta,L}}{\delta g(y)}(S^{\delta,L})^+\right\} &= 0 \qquad \text{when} \quad x \lesssim y \end{aligned} \tag{117}$$

[for details, see Alebastrov and Efimov (1973, 1974)].

The typical problem of nonlocal quantum field theory consists in proving the existence of the following sequence of limits in each perturbation order:

$$S^L = \lim_{\delta\Rightarrow 0} S^{\delta,L}, \qquad S = \lim_{L\Rightarrow\infty} S^L \tag{118}$$

Here the first limit $\delta \Rightarrow 0$ means the removal of all ghost states from the theory. The second limit $L \Rightarrow \infty$ means the passage to infinite volume and switching on the interaction over all four-dimensional space.

5.2.6. The Green Function in the Limit $\delta \Rightarrow 0$

The Green function in the limit $\delta \Rightarrow 0$ is the generalized function that is defined on a space of test functions Z_∞. Therefore, we have to consider improper transitions to the limit, i.e., investigate the limit

$$\lim_{\delta\Rightarrow 0}\int d^4x\, \Delta_c^\delta(x) f(x) = \lim_{\delta\Rightarrow 0}\int d^4k\, \tilde{\Delta}_c^\delta(k)\tilde{f}(k) \tag{119}$$

where $\Delta_c^\delta(x)$ is the Green function and $f(x) \in Z_\infty$. Omitting some details of the estimations (Efimov, 1977, 1985), one can obtain

$$\lim_{\delta\Rightarrow 0} \int d^4x\, \Delta_c^\delta(x) f(x)$$

$$= \lim_{\delta\Rightarrow 0} i^{-1}(2\pi)^{-4} \int d^4k\, \tilde{f}(k) \sum_{j=0}^{\infty} (-1)^j A_j(\delta)[m_j^2(\delta) - k^2 - i\epsilon]^{-1}$$

$$= \lim_{\delta\Rightarrow 0} i^{-1}(2\pi)^{-4} \int d^4k \frac{\tilde{f}(k)}{m^2 - k^2 - i\epsilon}$$

$$\times \sum_{n=0}^{\infty} \left\{ \frac{v_n r_0^{2n}(k^2 - m^2)^n}{\prod_{j=1}^{n+2}[1 - \delta m^{-2} j^{-\sigma}(k^2 - m^2 + i\epsilon)]} \right\}$$

$$= i^{-1}(2\pi)^{-4} \int d^4k\, V(-r_0^2 k^2) \tilde{f}(k)[m^2 - k^2 - i\epsilon]^{-1}$$

because of $f(x) \in Z_\infty$.

Thus the causal function $\Delta_c^\delta(x)$ changes in the limit $\delta \Rightarrow 0$ into a nonlocal confinement propagator:

$$\lim_{\delta\Rightarrow 0} \tilde{\Delta}_c^\delta(k) = \tilde{D}_c(k^2) = V_m(-r_0^2 k^2)[m^2 - k^2 - i\epsilon]^{-1} \tag{120}$$

The function $\tilde{D}_c(k^2)$ has no single poles at $k^2 = m^2$, in accordance with the function (59). However, the poles corresponding to all ghost states have disappeared.

The function (120) is the entire function and it does not correspond to any "real state." This function describes the nonlocal and confinement character of the interaction of our spread-out particles inside the hadron.

Thus in the limit $\delta \Rightarrow 0$ our theory becomes nonlocal. In other words, the "ghosts" (which disappear in the limit $\delta \Rightarrow 0$) as a self-memory make the theory nonlocal. Consequently, the nonlocal and confinement character of the interaction of the classical field (68) is revealed in quantum field theory as a residual effect of the nonphysical ghost states when these ghosts are removed by the transition to the limit $\delta \Rightarrow 0$.

6. NONLOCAL ELECTROMAGNETIC INTERACTIONS OF CONFINED QUARKS

6.1. Introduction

Before we construct a nonlocal quantum chromodynamics, we consider a simpler system: the interaction between a nonlocal photon and a confined quark. As shown above, in the case of the scalar particle the Lagrangian density (68) for the extended field $\Phi(x) = K_m(r_0^2\Box)\varphi(x)$ is equivalent to the usual standard form

$$L = \tfrac{1}{2}\varphi(x)(\Box - m^2)\varphi(x) + gU(K_m(r_0^2\Box)\varphi(x)) \tag{121}$$

with the nonlocal interaction term in accordance with the regularization procedure δ:

(a) The commutator $\Delta^\delta(x) = [\Phi^\delta(x), \Phi^\delta(0)]_-$ changes into the commutator of the scalar field $\varphi(x)$

$$\lim_{\delta\Rightarrow 0} \Delta^\delta(x) = \Delta(x) = (2\pi)^{-3}\int d^4k\, \epsilon(k_0)\delta(k^2 - m^2)e^{-ikx}$$

and

(b)

$$\lim_{\delta\Rightarrow 0} \Delta_\pm^\delta(x) = \Delta_\pm(x) = (2\pi)^{-3}\int d^4k\, \theta(\mp k_0)\delta(k^2 - m^2)e^{-ikx}$$

in the improper sense.

(c) The existence of these limits means that there exists a weak limit:

$$\lim_{\delta\Rightarrow 0} \Phi^\delta(x) = \varphi(x), \qquad (\Box - m^2)\varphi(x) = 0$$

(d) All ghost states disappear in the limit $\delta \Rightarrow 0$, i.e.,

$$\lim_{\delta\Rightarrow 0} H^\delta = H$$

Thus the initial Lagrangian describing the electromagnetic interaction of the quark may be chosen in the form

$$\mathcal{L}(x) = \mathcal{L}_0(x) + \mathcal{L}_{\text{in}}(x)$$

$$\mathcal{L}(x) = -\frac{1}{2} :[\partial_\nu A_\mu(x)][\partial_\nu A_\mu(x)]: + \sum_j :\bar{\Psi}_j(x)(i\hat{\partial} - m_j)\Psi_j(x):$$

$$\mathcal{L}_{\text{in}}(x) = e :\bar{\Psi}_R(x)\hat{A}_R(x)\Psi_R(x): \tag{122}$$

where $A_\nu^R(x)$ and $\Psi_j^R(x)$ are the nonlocal fields of photons and quarks. The summation in the expression $\mathcal{L}_0(x)$ extends over all quark fields ($j = u, d, c, \ldots$).

There are various approaches (Efimov, 1977; Namsrai, 1986; Moffat, 1990) to the gauge-invariant description of the nonlocal interaction of fermions (122). The theory with the interaction Lagrangian

$$\mathcal{L}_{\text{in}}^F(x) = e :\bar{\Psi}(x)\hat{A}_R(x)\Psi(x): \tag{123}$$

has been studied by Efimov (1977), where $\Psi(x)$ is the local spinor field. In this theory the pure fermion loops are not regularized in a unified way, so the vacuum polarization diverges.

The finiteness of these diagrams is achieved by using a special cyclic regularization procedure carried out over whole fermion loops [see Efimov (1977, 1985) for details].

Recently, the interaction Lagrangian

$$\mathscr{L}_{\text{in}}^{M}(x) = e :\bar{\Psi}_R(x)\hat{A}_R(x)\Psi_R(x) + \tilde{\mathscr{L}}_I \tag{124}$$

where $\tilde{\mathscr{L}}_I(\Psi, A_\mu)$ contains higher order interactions necessary to restore gauge invariance, has been studied by Moffat (1990). The third scheme (Namsrai, 1986) for the construction of the gauge-invariant nonlocal interaction of type (122) is based on the Feynman diagrammatic techniques, in which the gauge invariance of quark dynamics means that every matrix element of the S-matrix has a definite structure, and algebraic relations exist between them. For example, the following procedure (Namsrai, 1986) is based on Kroll (1966):

(1) To satisfy the conditions of gauge invariance for the nonlocal theory (with changing propagators of quark and photon), one must change the form of the one-photon vertex

$$\gamma_\mu \Rightarrow u_\mu(q, k) = -d_\mu(k)S_R^{-1}(\hat{q}) \tag{125}$$

due to the Ward–Takahashi identity

$$k_\mu \tilde{U}_\mu(p, q) = S_R(\hat{p}) - S_R(\hat{q}) \tag{126}$$

where

$$\tilde{U}_\mu(p, q) = S_R(\hat{p})u_\mu(q, k)S_R(\hat{q}) \qquad (p = k + q)$$

Here $S_R(\hat{p})$ is the nonlocal propagator of the quark and $d_\mu(k)$ is some operator whose actions on the entire functions are

$$d_\mu(k)V(-q^2r_0^2) = [V(-(q + k)^2r_0^2) - V(-q^2r_0^2)]\,\frac{\hat{k}\gamma_\mu}{k^2} \tag{127a}$$

$$d_\mu(k)V^{-1}(-q^2r_0^2) = -V^{-1}(-(q + k)^2r_0^2)[d_\mu(k)V(-q^2r_0^2)]V^{-1}(-q^2r_0^2) \tag{127b}$$

$$\begin{aligned} d_\mu(k)S_R(\hat{q}) &= -S_R(\hat{q} + \hat{k})[d_\mu(k)S_R^{-1}(\hat{q})]S_R(\hat{q}) \\ &= S_R(\hat{q} + \hat{k})\tilde{U}_{1\mu}(k, q)S_R(\hat{q}) \end{aligned} \tag{127c}$$

where

$$\tilde{U}_{1\mu}(k, q) = u_{1\mu}(k, q) = -d_\mu(k)S_R^{-1}(\hat{q})$$

(2) The proof of the validity of the nonlocal generalized Ward–Takahashi identity is

$$(p_\mu - q_\mu)\tilde{U}_\mu(p, q) = S_R(\hat{p}) - S_R(\hat{q}) \tag{128}$$

where

$$\tilde{U}_\mu(p, q) = S_R(\hat{p})u_\mu(q, k)S_R(\hat{q}), \qquad k = p - q$$

Taking into account the relation

$$u_\mu(q, k) = \gamma_\mu V^{-1}(-p^2r_0^2) + (m - \hat{q} - \hat{k})V^{-1}(-p^2r_0^2) \times [d_\mu(k)V(-q^2r_0^2)]V^{-1}(-q^2r_0^2) \tag{129}$$

and equation (127) we get, after some calculations, identity (128).

(3) The charged closed loop in the nonlocal theory (122) is determined by the expression

$$\tilde{\Pi}_n^R(k_1, \ldots, k_n) = \frac{1}{n}\int d^4q \, \mathrm{Tr}\{\tilde{U}_n^R(q; k_1, \ldots, k_n)S_R(\hat{q})\} \tag{130}$$

where

$$\tilde{U}_n^R(q; k_1, \ldots, k_n) = V(-q_n^2r_0^2)\tilde{U}_n(q; k_1, \ldots, k_n)$$

$$S_R(\hat{q}_n)\tilde{U}_n(q; k_1, \ldots, k_n)S_R(\hat{q}) = (-1)^n d(k_1) \cdots d(k_n)S_R(\hat{q}) \tag{131}$$

with $U_0 = S_R^{-1}(\hat{q})$. Tensor indexes are omitted here.

6.2. The Construction of the S-Matrix for the Quark Dynamics

Formally, with the interaction Lagrangian (122) the S-matrix can be written in the form of the T-products:

$$S = 1 + i\sum_{n=1}^{\infty}\frac{1}{n!}S_n$$

$$S_n = (i^n)^{-1}\int d^4x_1 \cdots \int d^4x_n \, T_d\left\{\prod_{j=1}^{n}\mathcal{L}_{\rm in}(x)\right\} \tag{132}$$

Here the symbol T_d means the so-called Wick T-product or T^*-operation (Bogolubov and Shirkov, 1980; Efimov, 1977) and the lowercase d corresponds to the algebraic prescription determined in Section 6.1. In order to construct the perturbation series for the S-matrix (132) by prescription of the usual local theory, it is necessary to change (in the Feynman diagrams)

$$\frac{m + \hat{k}}{m^2 - k^2 - i\epsilon} \Rightarrow \frac{m + \hat{k}}{m^2 - k^2 - i\epsilon}V_m(-k^2r_0^2)$$

$$g_{\mu\nu}(-k^2 - i\epsilon)^{-1} \Rightarrow g_{\mu\nu}V_0(-k^2r_0^2)(-k^2 - i\epsilon)^{-1} \tag{133}$$

and at the same time to insert the modified vertex (125) into the vertices of the external photon lines. The calculation of the matrix elements for the charged-quark loops will be undertaken using formulas (130) and (131).

For the purpose of calculation it is convenient to present the nonlocal form factor (59) in the Mellin representation

$$V_m^\delta(-p^2 r_0^2) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \, \frac{v(\xi)}{\sin \pi\xi} \, r_0^{2\xi} e^{\delta\xi^2}(m^2 - p^2 - i\epsilon)^\xi,$$

$$0 < \beta < 1 \tag{134}$$

where

$$v(\xi) = -\frac{\sqrt{\pi}}{\Gamma(\xi)} 2^{-2\xi-1}\left[\frac{\sqrt{\pi}}{\Gamma(2+\xi)} + \rho'^3 \frac{1}{\Gamma(5/2+\xi)}\right] \tag{135}$$

and the regularization multiplier $e^{\delta\xi^2}$ guarantees the passage to the Euclidean metric in intermediate calculations for the S-matrix elements.

6.2.1. The Diagrams of Vacuum Polarization

In the gauge-invariant nonlocal theory the vacuum polarization at the second order of perturbation theory (Fig. 5a) is given by an expression of type (130),

$$\Pi^R_{\mu\nu}(k_1, k_2) = \lim_{\delta\Rightarrow 0} \frac{ie^2}{2(2\pi)^4}\int d^4q \; V_m^\delta(-q^2 r_0^2)\,\mathrm{Tr}\{\tilde{U}^\delta_{\mu\nu}(q; k_1, k_2) S_R^\delta(\hat{q})\} \tag{136}$$

where $k_1 + k_2 = 0$,

$$S_R(\hat{q}_2)\tilde{U}_{\mu\nu}(q; k_1, k_2) S_R(\hat{q}) = (-1)^2 d_\mu(k_1) d_\nu(k_2) S_R(\hat{q}), \qquad q_2 = q + k_1 + k_2 = q$$

Expression (136) is simplified by the d-operation determined in Section 6.1.

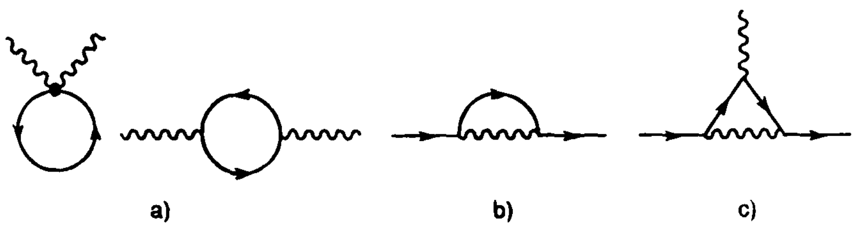

Fig. 5. The primitive Feynman diagrams in nonlocal quark dynamics.

Thus, making use of the definition of the d-operation for the entire functions and taking the trace of γ-matrices, we get

$$\begin{aligned}\tilde{\Pi}^r_{\mu\nu}(k) &= \lim_{\delta\Rightarrow 0}\frac{ie^2}{(2\pi)^4}\int d^4q\,\mathrm{Tr}\Big\{\gamma_\mu\frac{V^\delta_m(-q^2r_0^2)}{m-\hat{q}_1}\gamma_\nu\frac{1}{m-\hat{q}}+\gamma_\mu\frac{1}{m-\hat{q}_1}\\ &\quad\times[d_\mu(k)V^\delta_m(-q^2r_0^2)]\\ &\quad+\frac{1}{2}d_\mu(k)d_\nu(-k)V^\delta_m(-q^2r_0^2)\Big\}\\ &= \tilde{\Pi}^{(1)}_{\mu\nu}(k)+\tilde{\Pi}^{(2)}_{\mu\nu}(k)+\tilde{\Pi}^{(3)}_{\mu\nu}(k)\end{aligned}$$

where $q_1 = q + k$,

$$\begin{aligned}\tilde{\Pi}^{(1)}_{\mu\nu}(k) &= 4\rho(1-\xi)\int_0^1 dx\,(1-x)^{-\xi}\\ &\quad\times\int d^4q\,\mathcal{L}^{\xi-2}[2q_\mu q_\nu+2k_\mu q_\nu+g_{\mu\nu}(m^2-q^2-(kq))]\\ \tilde{\Pi}^{(2)}_{\mu\nu}(k) &= \frac{4\rho}{k^2}\int d^4q\,[q_\mu k_\nu-q_\nu k_\mu+g_{\mu\nu}(k^2+(kq))]\\ &\quad\times\left\{(m^2-q_1^2-i\epsilon)^{\xi-1}+\xi\int_0^1 dx\,(1-x)^{-1-\xi}\mathcal{L}^{\xi-1}\right\}\\ \tilde{\Pi}^{(3)}_{\mu\nu}(k) &= \frac{\rho}{k^4}(2k_\mu k_\nu-k^2g_{\mu\nu})\\ &\quad\times\int d^4q\,[(m^2-q^2-i\epsilon)^\xi-(m^2-q_1^2-i\epsilon)^\xi]\end{aligned}$$

Here we have used the representation (134) for the form factor and the notation

$$\begin{aligned}\mathcal{L} &= m^2-q^2-2x(kq)-k^2x\\ \rho &= \frac{ie^2}{(2\pi)^2}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\,\frac{v(\xi)}{\sin\pi\xi}\,r_0^{2\xi},\qquad 0<\beta<1\end{aligned}$$

Going to the Euclidean metric and integrating over d^4q, we obtain, in the limit $\delta \Rightarrow 0$,

$$\begin{aligned}\tilde{\Pi}^r_{\mu\nu}(k) &= \frac{e^2}{2\pi^2}(k_\mu k_\nu-k^2g_{\mu\nu})\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\,\frac{v(\xi)}{\sin\pi\xi}(m^2r_0^2)^\xi\\ &\quad\times\int_0^1 dx\,x(1-x)^{1-\xi}\frac{\Gamma(-\xi)}{\Gamma(1-\xi)}\mathcal{L}_0^\xi\\ \mathcal{L}_0 &= 1-\frac{k^2}{m^2}x(1-x)\end{aligned}$$

Assuming $m^2r_0^2 << 1$, we get

$$\tilde{\Pi}^r_{\mu\nu}(k) = \frac{e^2}{2\pi^2}(k_\mu k_\nu - k^2 g_{\mu\nu})\left\{\frac{\sqrt{\pi}}{12}\left[\sqrt{\pi} + \frac{4}{3}\frac{\rho'^3}{\sqrt{\pi}}\right] + \nu(1)m^2r_0^2\left[\frac{1}{2} - \frac{1}{12}\frac{k^2}{m^2}\right]\right\}$$

$$\nu(1) = -\frac{\sqrt{\pi}}{8}\left[\frac{\sqrt{\pi}}{2} + \frac{8}{15}\frac{\rho'^3}{\sqrt{\pi}}\right] \tag{137}$$

Here we have used the limit

$$\lim_{x \Rightarrow 0}\frac{\Gamma(-x)}{\Gamma(x)} = -1$$

and the fact that $\nu(0) = 0$. We observe that expression (137) is essentially different from the usual vacuum polarization in QED. Terms like $\ln(m^2r_0^2)$, which diverge at $r_0 \Rightarrow 0$, are absent. Therefore, the finite-charge renormalization is needed in the quark dynamics. It should be noted that in the limit $r_0 \Rightarrow 0$, our theory becomes trivial, i.e., all propagators of the quark and of the nonlocal "photon" become zero.

6.2.2. The Diagram of Self-Energy

The corresponding term in the S-matrix (Fig. 5b) can be written in the form

$$-i :\bar{\psi}(x)\Sigma_R(x-y)\psi(y):$$

$$\Sigma_R(x) = (2\pi)^{-2}\int d^4p\, e^{ipx}\tilde{\Sigma}_R(p)$$

where

$$\tilde{\Sigma}_R(p) = \lim_{\delta \Rightarrow 0}\frac{-ie^2}{(2\pi)^4}\int d^4k\,\frac{V_0^\delta(-k^2r_0^2)}{-k^2 - i\epsilon}\gamma_\mu$$

$$\times \frac{m + \hat{p} - \hat{k}}{m^2 - (p-k)^2 - i\epsilon}\gamma_\mu V_m^\delta(-(p-k)^2r_0^2)$$

The calculation of this expression is carried out in the same manner as in Namsrai (1986). The result reads

$$\tilde{\Sigma}_R(p) = \frac{e^2}{8\pi^2}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi\,\frac{\nu(\xi)}{\sin\pi\xi}(m^2r_0^2)^\xi\frac{1}{2i}$$

$$\times \int_{-\gamma+i\infty}^{-\gamma-i\infty} d\eta\,\frac{\nu(\eta)}{\sin\pi\eta}(m^2r_0^2)^\eta\frac{\Gamma(-\xi-\eta)}{\Gamma(1-\xi)\Gamma(1-\eta)}$$

$$\times \int_0^1 dx\,(1-x)^{\xi} x^{-\xi}\left(1-\frac{p^2}{m^2}x\right)^{\xi+\eta}(2m-\hat{p}x),$$

$$0<\beta<1, \quad 0<\gamma<1 \tag{138}$$

Assuming that the value of $m^2r_0^2$ is small, we obtain

$$\tilde{\Sigma}_R(p) = \frac{e^2}{8\pi^2}\frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi$$

$$\times \frac{v(\xi)}{\sin^2\pi\xi}\left\{v(-\xi)\left[2m-\frac{\hat{p}}{2}(1-\xi)\right] + m^2r_0^2 v(1-\xi)\xi\right.$$

$$\left.\times\left[2m-\frac{1}{2}\left(\hat{p}+2m\frac{p^2}{m^2}\right)(1-\xi)+\frac{1}{6}\frac{p^2}{m^2}\hat{p}^2(2-\xi)(1-\xi)\right]\right\}$$

We see again that this expression is finite at the limit $r_0^2 \Rightarrow 0$ and differs from the QED value.

If it is necessary for the choice of the function $v(\xi)$ in (135), it is simple to calculate integrals of the type

$$\frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi\,\frac{v(\xi)v(-\xi)}{\sin^2\pi\xi}(\cdots)$$

6.2.3. The Vertex Diagram and the Correction to the Anomalous Magnetic Moment of the Quark

In the momentum representation, the matrix element corresponding to the vertex diagram (Fig. 5c) has the standard form

$$\tilde{U}_\mu^R(p,q) = i^{-1}e^2(2\pi)^{-4}\int d^4k\, D_c^R(-(p-k)^2r_0^2)\gamma_\nu\, d_\mu(q)S_R(\hat{k})\gamma_\nu$$

where

$$d_\mu(q)S_R(\hat{k}) = (m-\hat{k}-\hat{q})^{-1}\gamma_\mu V_m(-k^2r_0^2)(m-\hat{k})^{-1}$$

$$+ (m-\hat{k}-\hat{q})^{-1}\{V_m(-(k+q^2)r_0^2) - V_m(-k^2r_0^2)\}\hat{q}\gamma_\mu q^{-2}$$

The symbol δ for the intermediate regularization procedure is omitted here. By using the identity

$$a^n - b^n = n\int_b^a dx\, x^{n-1} = n(a-b)\int_0^1 dx\,[(a-b)x+b]^{n-1}$$

we can transform the difference of the form factor values into

$$
\begin{aligned}
&V_m(-(k+q^2)r_0^2) - V_m(-k^2r_0^2) \\
&= -[q^2 + 2(kq)]\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{v(\xi)}{\sin \pi\xi}\xi r_0^{2\xi} \\
&\times \int_0^1 dx[m^2 - k^2 - 2x(kq) - q^2x]^{\xi-1}
\end{aligned}
$$

which is useful for concrete calculations.

The useful form for the matrix element of the vertex functions between two "free" single-quark states is

$$
\tilde{U}_\mu^R(p,q) = \bar{u}(p')\left\{\gamma_\mu F_1(q^2) + \frac{i}{2m}\sigma_{\mu\nu}q^\nu F_2(q^2)\right\}u(p) \tag{139}
$$

where

$$
p' = q + p, \qquad \sigma_{\mu\nu} = \frac{1}{2i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)
$$

$$
F_1(q^2) = f_1(q^2) + f_2(q^2), \qquad F_2(q^2) = g_1(q^2) + g_2(q^2)
$$

Here

$$
\begin{aligned}
f_1(q^2) &= N(\xi,\eta)\left\{\left[-2 + 8\beta - 2\beta^2 - 2\frac{q^2}{m^2}(1-\gamma)(1-\alpha)\right]\Gamma(1-\eta-\xi)\right. \\
&\quad \left.\times \mathcal{L}_1^{-1+\eta+\xi} - 2\Gamma(-\eta-\xi)\mathcal{L}_1^{\eta+\xi}\right\} \\
g_1(q^2) &= 4N(\xi,\eta)\beta(1-\beta)\Gamma(1-\eta-\xi)\mathcal{L}_1^{-1+\eta+\xi} \\
f_2(q^2) &= N(\xi,\eta)\eta\int_0^1 dt\left\{\left[2\frac{q^2}{m^2} - 2\frac{q^2}{m^2}(\alpha + t\gamma)\right](1-\beta-2\alpha-2t\gamma)\right. \\
&\quad \left.\times \Gamma(1-\eta-\xi)\mathcal{L}_1^{-1+\eta+\xi} - 2\Gamma(-\eta-\xi)\mathcal{L}_2^{\xi+\eta}\right\} \\
g_2(q^2) &= 4N(\xi,\eta)\eta\int_0^1 dt\,(1-\beta-2\alpha-2t\gamma)\Gamma(1-\eta-\xi)\mathcal{L}_2^{-1+\eta+\xi} \\
N(\xi,\eta) &= \frac{e^2}{(2\pi)^4}\frac{\pi^2}{2i}\int_{-\delta+i\infty}^{-\delta-i\infty} d\xi\frac{v(\xi)}{\sin\pi\xi}(m^2r_0^2)^\xi \frac{1}{2i}\int_{-\rho+i\infty}^{-\rho-i\infty} d\eta\frac{v(\eta)}{\sin\pi\eta}(m^2r_0^2)^\eta
\end{aligned}
$$

$$\times \frac{1}{\Gamma(1-\xi)\Gamma(1-\eta)} \int \int_0^1 \int d\alpha\, d\beta\, d\gamma\, \beta^{-\xi}\gamma^{-\eta}\delta(1-\alpha-\beta-\gamma)$$

$$\mathcal{L}_1(q^2) = (1-\beta)^2 - (q^2/m^2)\alpha\gamma$$

$$\mathcal{L}_2(q^2) = \mathcal{L}_1(q^2) - (q^2/m^2)t\gamma(1-t\gamma) + 2(q^2/m^2)t\alpha\gamma + (q^2/m^2)t\gamma\beta$$

Ultraviolet and infrared divergent terms in the first expression of (139) in the limit $q^2 \Rightarrow 0$ are absent, and therefore the corresponding charge renormalization of the quarks is finite. The second term (139) at $q^2 = 0$ contributes to the anomalous magnetic moment of the quarks by

$$a_j = F_2(0) = \frac{4}{(2i)^2}\frac{\alpha}{2\pi} \int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \int_{-\gamma+i\infty}^{-\gamma-i\infty} d\eta \frac{v(\xi)v(\eta)}{\sin\pi\xi \sin\pi\eta} (m^2 r_0^2)^{\xi+\eta}$$

$$\times (1-\eta)(1-\xi) \frac{\Gamma(1-\eta-\xi)\Gamma(1+2\xi+\eta)}{\Gamma(3-\eta)\Gamma(3+\eta+\xi)},$$

$$0 < \beta, \gamma < 1 \tag{140}$$

As usual, assuming $m^2 r_0^2 << 1$, one gets

$$a_j = \frac{\alpha}{2\pi}\left\{m^2 r_0^2\left[-\frac{2}{3}\frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{v(\xi)v(1-\xi)}{\sin^2\pi\xi}\xi(1-\xi)\right]\right\} \tag{141}$$

We see that the Schwinger term $\alpha/2\pi$ is absent here. This is natural, because our theory does not contain the usual local electrodynamics of the quarks in the limit $r_0^2 \Rightarrow 0$.

Thus the S-matrix (132) is gauge invariant. Indeed, in the quark dynamics under consideration the Ward identity

$$\frac{\partial}{\partial p_\mu} \tilde{\Sigma}_R = -\tilde{U}_\mu^R(p, 0) \tag{142}$$

is valid, since this identity is a direct consequence of the identities (127) and (125). Since it is not necessary to subtract any infinite counterterms, no dangerous terms which can break the Ward identity (142) when formula (127) is valid will appear in the perturbation theory. The diagram of the vacuum polarization is gauge invariant due to our choice of the gauge-invariant regularization procedure of Kroll (1966).

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